

The Asymmetric CFT Landscape in $D = 4, 6, 8$ with Extended Supersymmetry

Ralph Blumenhagen¹, Michael Fuchs¹, Erik Plauschinn²

¹ *Max-Planck-Institut für Physik (Werner-Heisenberg-Institut),
Föhringer Ring 6, 80805 München, Germany*

² *Arnold Sommerfeld Center for Theoretical Physics,
LMU, Theresienstr. 37, 80333 München, Germany*

Abstract

We study asymmetric simple-current extensions of Gepner models in dimensions $D = 4, 6, 8$ with at least eight supercharges in the right-moving sector. The models obtained in an extensive stochastic computer search belong to a small number of different classes. These classes can be categorized as dimensional reductions, asymmetric orbifolds with $(-1)^{F_L}$, extra gauge enhancement and as coming from the super Higgs-effect. Models in the latter class are particularly interesting, as they may correspond to non-geometric flux compactifications.

Contents

1	Introduction	2
2	The ACFT construction	4
2.1	Simple current extension	5
2.2	Review of Gepner construction	6
2.3	Asymmetric Gepner Models	8
3	The landscape of ACFTs	8
3.1	Classification scheme	9
3.2	Super Higgs effect	9
3.3	Asymmetric $(-1)^{F_L}$ shift orbifolds	10
3.4	ACFTs in $D = 8$	12
3.5	ACFTs in $D = 6$	14
3.6	ACFTs in $D = 4$	18
4	Conclusions	24
A	Supermultiplets	27
A.1	Supergravity in $D = 8$	27
A.2	Supergravity in $D = 6$	29
A.3	Supergravity in $D = 4$	31

1 Introduction

Much of the work on string compactifications is concerned with the description and understanding of geometric backgrounds. Examples are type II and heterotic models compactified on Calabi-Yau manifolds, left-right symmetric orbifolds and orientifolds thereof. However, from the early days of string theory on it has been clear that also fully consistent *left-right asymmetric* conformal field theories (ACFTs) exist. The most prominent examples are asymmetric orbifolds [1] and free-fermion constructions [2–4], and for a discussion of D-branes in asymmetric backgrounds see [5–8]. More recently it has become clear that at least some of these asymmetric constructions correspond to NS-NS flux compactifications, in particular, they correspond to Minkowski minima of gauged supergravity (GSUGRA) theories with spontaneously-, partially-broken supersymmetry [9–16]. The general GSUGRA theory involves non-geometric fluxes that naturally appear in double field theory, which is a proposed field theory that features manifest $O(D, D)$ invariance (for reviews see [17–19]). However, it is fair to say that the constraints on such non-geometric fluxes are not yet fully understood, i.e. it is not clear which minima of GSUGRA belong to the string landscape and which to the swampland.

This work can be considered as the continuation of a series of papers that started with the construction of Gepner models [20–23], their extension by the simple current technique [24, 25] for constructing ACFTs [26–30], and the recent attempt [16] to relate some of these four-dimensional ACFTs to minima of $\mathcal{N} = 2$ GSUGRA partially-broken to $\mathcal{N} = 1$.¹ For instance, it was proposed that the $\mathbf{k} = (3^5)$ Gepner model, extended by a certain asymmetric simple current, corresponds to a Minkowski minimum of a non-geometric flux compactification on the complete intersection Calabi-Yau threefold $\mathbb{P}_{1,1,1,1,2,2}$ [5, 3]. The latter identification was impeded by the existence of a superpotential in four-dimensional $\mathcal{N} = 1$ theories, due to which an arbitrary number of chiral fields can become massive. For this reason, it is desirable to investigate the analogous ACFT-GSUGRA correspondence in a simpler setting, where supersymmetry is more strongly protecting the generation of mass terms.

Therefore, in this paper we consider ACFTs in the framework of the type IIB superstring theory in dimensions $D = 4, 6, 8$, constructed by extensions via simple currents with at least *eight* supercharges arising from the right-moving sector. Thus, the models have at least $\mathcal{N} = 2$ in 4D or $\mathcal{N} = 1$ in 6D and 8D. Geometric compactifications belonging to these classes are on \mathbb{T}^2 , $K3$ and $K3 \times \mathbb{T}^2$, respectively. However, we also found models with e.g. $\mathcal{N} = 5$ or $\mathcal{N} = 3$ supersymmetry in 4D that clearly can only appear in asymmetric constructions. Since we are working with models with extended supersymmetry, we expect that the identification of these abstractly defined ACFTs is simpler than in our earlier work [16]. This indeed turns out to be the case in an unprecedented clarity.

Through an extensive stochastic computer search, we have constructed hundreds of millions of different ACFTs and found that they can be understood via four different mechanisms. In essence, our classification exploits the fact that in theories with eight supercharges, no generic scalar potential exists and therefore modes can become massive only via a Higgs mechanism. The four mechanisms are characterized as follows:

- First, some of the lower-dimensional models are simply dimensional reductions of higher-dimensional models.
- Second, as it often happens for Gepner models, there can be special gauge enhancements that can be Higgsed.
- Third, even though R-R fluxes are not expected to be visible in a CFT, it turns out that the asymmetric operation $(-1)^{F_L}$ can be realized via simple currents. Here F_L is the left-moving space-time fermion number, that is even for the left-moving NS-sector and odd for the R-sector. In all dimensions we found a class of models that can be identified with asymmetric

¹Similar asymmetric simple current extensions in the context of the heterotic string were discussed in [31–33].

$(-1)^{F_L}$ shift orbifolds with non-abelian gauge symmetries. Such models were already considered mostly in 4D in [34–36].

- Finally and most importantly, the most extensive series of models we found can be understood via the super Higgs mechanism [37–40]. This mechanism determines how a supergravity theory with \mathcal{N}' supersymmetries can be broken to a theory with $\mathcal{N} < \mathcal{N}'$ supersymmetries, consistent with the multiplet structure of both theories. This puts a number of constraints on the massless spectra of \mathcal{N} -supergravity models, which can be considered as necessary conditions for the gauged \mathcal{N}' -supergravity theory to admit Minkowski minima with \mathcal{N} -supersymmetry. For extended supersymmetries in 4D this was discussed in some detail in [40]. Some of these models can also be realized as asymmetric orbifolds that involve left-right asymmetric discrete symmetries but no $(-1)^{F_L}$ factor.

Exploiting these four mechanisms, we are able to provide a classification of all ACFT models found in our computer search. The emerging picture is quite compelling, but due to the stochastic nature of our search we cannot claim completeness of the ACFT landscape. In particular, there can exist islands [41] which cannot be reached via the simple-current technique. Moreover, as will be discussed, in cases where a super Higgs mechanism can work, i.e. where Minkowski type flux vacua can exist in principle, fairly large classes of ACFTs do appear. We are not yet at the stage where we can provide a one-to-one correspondence between asymmetric CFT data and concrete gaugings or fluxes, but the results look encouraging.

This paper is organized as follows: In section 2 we briefly introduce Gepner models and their simple current extensions. Section 3 is the main section and contains the presentation and classification of the ACFTs we found in our stochastic search. For each class we only present a typical representative example. More details on these models can be found at the URL-link [42]. Section 4 contains our conclusions, and the appendix contains an overview of multiplets in extended supersymmetries in $D = 4, 6, 8$ dimensions.

2 The ACFT construction

In this section, we briefly review the asymmetric conformal field theory construction employed in this paper. This is meant to explain our procedure to find models, the notation and the adjustments one has to make when working in different dimensions. For a more detailed explanation we would like to refer the reader for instance to [43] and to our previous paper [16].

2.1 Simple current extension

In many rational conformal field theories there exist primary fields J_a called simple currents [24, 25], whose fusion with any primary ϕ_i gives exactly one primary field, i.e.

$$J_a \times \phi_i = \phi_{J(i)}. \quad (2.1)$$

By associativity it is easy to see that the fusion of two simple currents is a simple current as well. Furthermore, having finitely many primaries it is clear that $J_a^{\mathcal{N}_a} = 1$ for a certain length \mathcal{N}_a . As a consequence the simple currents group the primaries into orbits $\{\phi_i, J_a \times \phi_i, J_a^2 \times \phi_i, \dots, J_a^{\mathcal{N}_a-1} \times \phi_i\}$. The above fusion rules result in the operator product expansion

$$J_a(z) \phi_i(w) = (z - w)^{-Q_i^{(a)}} \phi_{J(i)}(w) + \dots, \quad (2.2)$$

where $Q_i^{(a)}$ is called the monodromy charge. Using $J_a^{\mathcal{N}_a} = 1$ in this OPE one easily finds $Q_i^{(a)} = \frac{t_a^i}{\mathcal{N}_a} \bmod 1$ with an integer t_a^i . Furthermore, applying the simple current to an arbitrary OPE shows that the monodromy charge is a conserved quantity. If a so called monodromy parameter is even (for details consult e.g. [16, 24, 25, 43]), a simple current implies the existence of the following off-diagonal modular invariant partition function

$$Z_a(\tau, \bar{\tau}) = \vec{\chi}^T(\tau) M(J_a) \vec{\chi}(\bar{\tau}) = \sum_{k,l} \chi_k(\tau) (M_a)_{kl} \chi_l(\bar{\tau}), \quad (2.3)$$

where

$$(M_a)_{kl} = \sum_{p=1}^{\mathcal{N}_a} \delta(\phi_k, J_a^p \times \phi_l) \delta^{(1)}(\hat{Q}^{(a)}(\phi_k) + \hat{Q}^{(a)}(\phi_l)) \quad (2.4)$$

with

$$\hat{Q}^{(a)}(\phi_i) = \frac{t_a^i}{2\mathcal{N}_a} \bmod 1. \quad (2.5)$$

Notice that the simple current takes the primaries of the left side and couples them to their whole orbit (if the monodromy charges fit). Of course the combination of several modular matrices like $Z_{a_1, a_2} = \frac{1}{N} \sum_{k,l,m} \chi_l (M_{a_1})_{lk} (M_{a_2})_{km} \chi_m$ is also a modular invariant. N ensures the correct normalization of the vacuum. If two simple currents are relatively local, that is $Q^{(a_1)}(J_{a_2}) = 0$, the matrices M_1 and M_2 commute.

2.2 Review of Gepner construction

For $c < 3$ one finds only a discrete set of unitary $N = 2$ superconformal field theories (SCFTs), called minimal models, whose central charge

$$c = \frac{3k}{k+2} \quad (2.6)$$

is parametrized by the level $k = 1, 2, \dots$. The primaries are labeled by three integer quantum numbers (l, m, s) in the range

$$l = 0, \dots, k, \quad m = -k-1, -k, \dots, k+2, \quad s = -1, 0, 1, 2, \quad (2.7)$$

where $l+m+s$ must be even. Additionally one needs to impose the identifications

$$(l, m, s) \sim (k-l, m+k+2, s+2), \quad s \sim s+4, \quad m \sim 2(k+2). \quad (2.8)$$

For $s = 0, 2$ the state is in the Neveu-Schwarz (NS) sector, for $s = -1, 1$ the state is in the Ramond (R) sector. To compute the conformal dimension and the charge one needs to bring (l, m, s) into the standard range $|m-s| \leq l$ using (2.8). Then

$$\begin{aligned} \Delta_{m,s}^l &= \frac{l(l+2) - m^2}{4(k+2)} + \frac{s^2}{8}, \\ q_{m,s}^l &= \frac{m}{(k+2)} - \frac{s}{2}. \end{aligned} \quad (2.9)$$

Every primary with $l = 0$ is a simple current, whose fusion with another primary reads

$$\phi_{(m_1, s_1)}^0 \times \phi_{(m_2, s_2)}^{l_2} = \phi_{(m_1+m_2, s_1+s_2)}^{l_2}. \quad (2.10)$$

Gepner's construction uses tensor products of these minimal models $\bigotimes_{i=1}^r (k_i)$ as the internal CFTs of a type II compactification with d compact internal directions and D extended external directions. Depending on the desired internal dimensions $d = 2, 4, 6$ the central charges of the minimal models must therefore add up to $c_{\text{int}} = 3, 6, 9$.

Using light-cone gauge to eliminate two dimensions, we have $D-2 = 6, 4, 2$ non-compact directions and in turn $c_{\text{ext}} = 9, 6, 3$. For the external CFT one takes $D-2$ free bosons with $c_{\text{Bos}} = D-2$. Their superpartners are $D-2$ free fermions transforming in the vector representation of the little group $SO(D-2)$. Their symmetry algebra is therefore the $\widehat{\mathfrak{so}}(D-2)_1$ Kac-Moody algebra with $c_{\text{Ferm}} = \frac{D-2}{2}$. The four irreducible representations of $\widehat{\mathfrak{so}}(D-2)_1$ are (c, o, s, v) labeled by $s_0 = -1, 0, 1, 2$. Using $n = \frac{D-2}{2}$, their conformal weight, charge and degeneracy are

character	h	$q \bmod 2$	degeneracy
$\chi_o = \frac{1}{2} \left(\left(\frac{\theta_3}{\eta} \right)^n + \left(\frac{\theta_4}{\eta} \right)^n \right)$	0	0	0
$\chi_v = \frac{1}{2} \left(\left(\frac{\theta_3}{\eta} \right)^n - \left(\frac{\theta_4}{\eta} \right)^n \right)$	$\frac{1}{2}$	1	$2n$
$\chi_s = \frac{1}{2} \left(\left(\frac{\theta_2}{\eta} \right)^n + \left(\frac{\theta_1}{\eta} \right)^n \right)$	$\frac{n}{8}$	$\frac{n}{2}$	2^{n-1}
$\chi_c = \frac{1}{2} \left(\left(\frac{\theta_2}{\eta} \right)^n - \left(\frac{\theta_1}{\eta} \right)^n \right)$	$\frac{n}{8}$	$\frac{n}{2} - 1$	2^{n-1}

From the fusion rules

n odd	o	v	s	c	n even	o	v	s	c
o	o	v	s	c	o	o	v	s	c
v	v	o	c	s	v	v	o	c	s
s	s	c	v	o	s	s	c	o	v
c	c	s	o	v	c	c	s	v	o

one sees that all primaries are simple currents. To summarize, a state in a Gepner model reads

$$(l_1 \ m_1 \ s_1) \dots (l_r \ m_r \ s_r)(s_0) \in \bigotimes_{i=1}^r (k_i) \otimes \widehat{\mathfrak{so}}(D-2)_1. \quad (2.11)$$

To construct a fully positive partition function one starts with a purely bosonic CFT with $c = 24$ which is mapped to a SCFT by the bosonic string map relating $\widehat{\mathfrak{so}}(D-2)_1 \rightarrow \widehat{\mathfrak{so}}(D+6)_1 \otimes (E_8)_1$ via

$$\phi_{\text{bsm}}(\chi_o, \chi_v, \chi_s, \chi_c) \rightarrow (\chi_v, \chi_o, -\chi_c, -\chi_s) \otimes 1. \quad (2.12)$$

Notice that the difference of the conformal dimension of the characters in $\widehat{\mathfrak{so}}(D-2)_1$ and $\widehat{\mathfrak{so}}(D+6)_1$ is $\frac{1}{2}$ such that level matched states stay level matched under the bosonic string map. We also need the relatively local simple currents

$$J_{\text{GSO}} = (0 \ 1 \ 1) \dots (0 \ 1 \ 1)(s),$$

$$J_i = (0 \ 0 \ 0) \dots \underbrace{(0 \ 0 \ 2)}_{i^{\text{th}}} \dots (0 \ 0 \ 0)(v). \quad (2.13)$$

J_{GSO} implements the GSO-projection while the J_i ensure that NS and R sectors are not mixed in a state. The total partition function is

$$Z_{\text{Gepner}}(\tau, \bar{\tau}) \sim \bar{\chi}^T(\tau) M(J_{\text{GSO}}) \prod_{i=1}^r M(J_i) \bar{\chi}(\bar{\tau}) \Big|_{\phi_{\text{bsm}}^{-1}}, \quad (2.14)$$

where the bosonic string map has to be applied at the end and we neglected the contribution from the free bosons and possible normalization factors. This partition function allows us to read off the massless spectrum of the theory.

2.3 Asymmetric Gepner Models

In the following we analyze Gepner models when adding one or more possibly left-right asymmetric simple currents J_1, \dots, J_n

$$Z_{\text{ACFT}}(\tau, \bar{\tau}) \sim \bar{\chi}^T(\tau) M(J_1) \dots M(J_n) M(J_{\text{GSO}}) \prod_{i=1}^r M(J_i) \bar{\chi}(\bar{\tau}) \Big|_{\phi_{\text{bsm}}^{-1}}. \quad (2.15)$$

The additional simple currents often break or enhance supersymmetry. As we want to find the ACFTs corresponding to supersymmetry breaking of a GSUGRA with more than 8 supercharges we often need to enhance supersymmetry towards our desired starting point. Afterwards the left-moving supersymmetry is broken by left-right asymmetric simple currents. In the partition function this requires to correctly order the enhancing and breaking simple currents

$$Z_{\text{ACFT}}(\tau, \bar{\tau}) \sim \bar{\chi}^T(\tau) M(J_{\text{break}}) M(J_{\text{enhance}}) M(J_{\text{GSO}}) \prod_{i=1}^r M(J_i) \bar{\chi}(\bar{\tau}) \Big|_{\phi_{\text{bsm}}^{-1}}. \quad (2.16)$$

Note that we sometimes use more than one breaking and enhancing simple current.

Let us give two four-dimensional examples for this procedure: The Gepner model $\mathbf{k} = (1, 3, 3, 4, 8)$ has the standard $\mathcal{N} = \mathcal{N}_L + \mathcal{N}_R = 1 + 1$ supersymmetry. Taking the D-invariant in the last factor using the simple current

$$J_{\text{enhance}} = J_{\text{D}} = (0 \ 0 \ 0)(0 \ 0 \ 0)(0 \ 0 \ 0)(0 \ 0 \ 0)(0 \ 10 \ 2)(o), \quad (2.17)$$

supersymmetry is enhanced to $\mathcal{N} = \mathcal{N}_L + \mathcal{N}_R = 2 + 2$. This corresponds to the well known $K3 \times \mathbb{T}^2$ compactification. Further left-right asymmetric simple currents can break the left moving supersymmetry down to $\mathcal{N}_L \in \{1, 0\}$. As such we get models with total supersymmetry $\mathcal{N} \in \{3, 2\}$. A second example we will present later uses simple currents giving the \mathbb{T}^6 compactification with $\mathcal{N} = \mathcal{N}_L + \mathcal{N}_R = 4 + 4$. Further simple currents allow us to break the left moving supersymmetry down to $\mathcal{N}_L \in \{2, 1, 0\}$. The resulting models have therefore the total supersymmetry $\mathcal{N} \in \{6, 5, 4\}$.

3 The landscape of ACFTs

In this section, we present the results of a scan over eight-, six- and four-dimensional ACFTs which feature at least two supersymmetries arising from the right-moving sector. In the framework of asymmetric simple current extensions of Gepner models we constructed of the order of 10^8 different modular invariant partition functions and evaluated their massless spectra. It turned out that they all fall into a few different classes, that provide a natural classification scheme

for all these models. Remarkably, even though the construction is rather abstract and CFT based, these classes show some nice patterns that we make an attempt to understand from a broader perspective, i.e. by utilizing relations following from dimensional reduction, gauged supergravity and toroidal orbifolds. Before we present the results of our ACFT landscape study, let us introduce our classification scheme and a few structures that will become important.

3.1 Classification scheme

Our starting points are Gepner models corresponding to \mathbb{T}^2 , $K3$ and $K3 \times \mathbb{T}^2$ compactifications, and we introduce a classification scheme ${}^D\mathfrak{N}_{[\mathcal{N}_L, \mathcal{N}_R]}$ where D denotes the number of uncompactified dimensions and $\mathcal{N}_{L/R}$ the number of supersymmetries arising in the ACFT from the left and right moving sector, respectively. As mentioned, we only consider models with $\mathcal{N}_R \geq 2$. The number of supercharges is then given by $Q = 2^{\frac{D}{2}} (\mathcal{N}_L + \mathcal{N}_R)$.

Moreover, we present the massless spectrum for some representative ACFTs by their raw data, i.e. we provide for a massless sector the number of massless states transforming in the four representations (v, s, c, o) of the left- and right-moving little groups $SO(D-2)$. Each such sector fills out massless supermultiplets of $\mathcal{N} = \mathcal{N}_L + \mathcal{N}_R$ supersymmetry in D -dimensions. Thus, we present the data in the form

$${}^D\mathfrak{N}_{[\mathcal{N}_L, \mathcal{N}_R]} : \begin{cases} (n_v^{(0)}, n_s^{(0)}, n_c^{(0)}, n_o^{(0)})_L \otimes (n_v^{(0)}, n_s^{(0)}, n_c^{(0)}, n_o^{(0)})_R & \text{supermultipl.} \\ (n_v^{(c)}, n_s^{(c)}, n_c^{(c)}, n_o^{(c)})_L \otimes (n_v^{(c)}, n_s^{(c)}, n_c^{(c)}, n_o^{(c)})_R & \text{supermultipl.} \\ \dots & \dots \end{cases}$$

where, as indicated by the superscript, we state the results from the charged and the vacuum sector separately. Note that for each class, we have found a large number of concrete realizations, while here we only present some typical representatives.

3.2 Super Higgs effect

To order the classes of models we found, we use some non-CFT structures. In general the models are left-right asymmetric, i.e they do not correspond to purely geometric backgrounds. In particular, non-geometric NS-NS flux compactifications are in general expected to be described by ACFTs. Turning on such fluxes on e.g. \mathbb{T}^4 , $K3$ or $K3 \times \mathbb{T}^2$ leads to gauged supergravity theories. Of course it is difficult to identify directly the gaugings or fluxes from an ACFT model, but there are prerequisites for a gauging of a SUGRA theory with \mathcal{N}' supersymmetries to admit a Minkowski vacuum with \mathcal{N} supersymmetries. There must be the super Higgs mechanism at work [37–40].

Let us recall this for the four-dimensional example of $\mathcal{N}' = 8$ to $\mathcal{N} = 6$ breaking. Ungauged $\mathcal{N}' = 8$ corresponds to the low energy effective field theory of type II compactified on a \mathbb{T}^6 . All massless states fit into the supergravity multiplet (see appendix A.3 for our notation and more details)

$$\begin{aligned} \text{massless} \quad \mathcal{G}_{(8)} &= 1 \cdot [2] + 8 \cdot [\tfrac{3}{2}] + 28 \cdot [1] + 56 \cdot [\tfrac{1}{2}] + 70 \cdot [0] \\ &= (2)_{\text{b}} + (16)_{\text{f}} + (56)_{\text{b}} + (112)_{\text{f}} + (70)_{\text{b}}. \end{aligned} \quad (3.1)$$

The numbers in the second line denote the number of on-shell degrees of freedom. If there exists a flux that breaks $\mathcal{N}' = 8$ to $\mathcal{N} = 6$, two gravitinos must become massive and become part of a massive spin-3/2 supermultiplet of $\mathcal{N} = 6$ supergravity. Moreover, the remaining degrees of freedom must fit into massless $\mathcal{N} = 6$ supermultiplets. The massless supergravity multiplet of $\mathcal{N} = 6$ reads

$$\begin{aligned} \text{massless} \quad \mathcal{G}_{(6)} &= 1 \cdot [2] + 6 \cdot [\tfrac{3}{2}] + 16 \cdot [1] + 26 \cdot [\tfrac{1}{2}] + 30 \cdot [0] \\ &= (2)_{\text{b}} + (12)_{\text{f}} + (32)_{\text{b}} + (52)_{\text{f}} + (30)_{\text{b}}, \end{aligned} \quad (3.2)$$

and the massive spin-3/2 supermultiplet is given by

$$\begin{aligned} \text{massive} \quad \overline{\mathcal{S}}_{(6)} &= 2 \cdot \left([\tfrac{3}{2}] + 6 \cdot [1] + 14 \cdot [\tfrac{1}{2}] + 14 \cdot [0] \right) \\ &= (8)_{\text{f}} + (36)_{\text{b}} + (56)_{\text{f}} + (28)_{\text{b}}. \end{aligned} \quad (3.3)$$

This is a $\frac{1}{2}$ -BPS short multiplet so that for CPT invariance it comes in a pair. Thus we see that the number of bosonic and fermionic degrees of freedom perfectly match to allow that the massless $\mathcal{N}' = 8$ gravity multiplet splits into the $\mathcal{N} = 6$ gravity multiplet plus a pair of massive spin-3/2 supermultiplets. As a consequence kinematically the super Higgs effect is possible, which is a necessary condition for the existence of an $\mathcal{N} = 6$ Minkowski minimum in $\mathcal{N}' = 8$ GSUGRA. When analyzing the ACFT results, we will often employ analogous super Higgs effects and its predictions on the remaining massless spectrum.

3.3 Asymmetric $(-1)^{F_L}$ shift orbifolds

However, not all ACFT models in our search will admit an interpretation in terms of a super Higgs effect. Even though Ramond-Ramond fluxes are expected not to be present in the ACFTs, we find that asymmetric orbifolds involving $(-1)^{F_L}$ can be realized via simple currents. Here F_L denotes the left-moving space-time fermion number, i.e. states in the left-moving NS-sector are even and states in the left-moving R-sector are odd. Such models were of interest for the appearance of perturbative non-abelian gauge symmetries for the type II superstring theories [34, 35]. Let us present a simple example in 8D that will also appear among the ACFTs models.

We start with type IIB compactified on the rectangular \mathbb{T}^2 at self-dual radii $r_i = \sqrt{\alpha'}$, with $i = 1, 2$, for the two circles. The partition function for the simple toroidal compactification can then be written as

$$Z_{\mathbb{T}^2} = (V_8 - S_8)(\tau) (\overline{V}_8 - \overline{S}_8)(\overline{\tau}) \Lambda_{\vec{m}, \vec{n}}^{(2)}(\tau, \overline{\tau}), \quad (3.4)$$

with the contribution from the Kaluza-Klein and winding modes (at $r_i = \sqrt{\alpha'}$)

$$\Lambda_{\vec{m}, \vec{n}}^{(2)}(\tau, \overline{\tau}) = \sum_{\vec{m}, \vec{n} \in \mathbb{Z}^2} q^{\frac{1}{4} \sum_i (m_i - n_i)^2} \overline{q}^{\frac{1}{4} \sum_i (m_i + n_i)^2}. \quad (3.5)$$

Here O_8, V_8, S_8, C_8 denote the characters of the four conjugacy classes of the $SO(8)_1$ Kac-Moody algebra from page 7 and we have skipped the contribution $1/|\eta|^{16}$ from the eight bosons. Now we define an asymmetric orbifold

$$\mathcal{A}_8 = \frac{\mathbb{T}^2}{(-1)^{F_L} S W}. \quad (3.6)$$

The orbifold projection $(-1)^{F_L}$ eliminates all states from the left-moving Ramond sector. Moreover, S denotes the momentum shift and W the winding shift along both directions of \mathbb{T}^2 . These two act on the momentum- and winding-modes as

$$S : (-1)^{\sum_i m_i} =: (-1)^{\vec{m}}, \quad W : (-1)^{\sum_i n_i} =: (-1)^{\vec{n}}. \quad (3.7)$$

It is straightforward to compute the partition function of this asymmetric orbifold as²

$$\begin{aligned} Z_{\text{ACFT}} = \frac{1}{2} \left[\right. & (V_8 - S_8)(\overline{V}_8 - \overline{S}_8) \Lambda_{\vec{m}, \vec{n}}^{(2)} \\ & + (V_8 - S_8)(\overline{V}_8 + \overline{S}_8) (-1)^{\vec{m} + \vec{n}} \Lambda_{\vec{m}, \vec{n}}^{(2)} \\ & + (V_8 - S_8)(\overline{O}_8 - \overline{C}_8) \Lambda_{\vec{m} + \frac{\vec{1}}{2}, \vec{n} + \frac{\vec{1}}{2}}^{(2)} \\ & \left. + (V_8 - S_8)(\overline{O}_8 + \overline{C}_8) (-1)^{\vec{m} + \vec{n}} \Lambda_{\vec{m} + \frac{\vec{1}}{2}, \vec{n} + \frac{\vec{1}}{2}}^{(2)} \right]. \end{aligned} \quad (3.8)$$

The first two lines correspond to the untwisted sector and the last two to the twisted sector. Note that taking a momentum- or winding-shift along a single S^1 , the corresponding partition function does not satisfy the level matching condition.

The resulting massless spectrum can be read off from (3.8). In the untwisted sector there are 64 bosonic and fermionic modes that combine into the $\mathcal{N} = 1$ supergravity multiplet plus two vectormultiplets (see appendix A.1 for details). From the twisted sector, $V_8 \overline{O}_8$ can combine with states from

$$\Lambda_{\vec{m} + \frac{\vec{1}}{2}, \vec{m} + \frac{\vec{1}}{2}}^{(2)} = q^0 \sum_{\vec{m}} \overline{q}^{\frac{1}{4} \sum_i (2m_i + 1)^2} \quad (3.9)$$

²We use the notation of [44].

to form a level matched massless state. Namely, the four combinations $m_1, m_2 \in \{0, -1\}$ give rise to four $\mathcal{N} = 1$ vectormultiplets. These four states provide the W -bosons of an $SU(2) \times SU(2)$ non-abelian gauge group. Its Coloumb-branch corresponds to changing the two radii of the \mathbb{T}^2 .

Such a construction can be generalized to 6D and 4D, where one also combines $(-1)^{F_L}$ with a left-moving shift of the Narain lattice of the tori. Such models have been classified in 4D in [34] with the result that, starting with the D_6 -lattice, one gets a model with $[0, 4]$ supersymmetry and maximal gauge symmetry $SU(2)^6$. Analogously, in 6D one gets $[0, 2]$ supersymmetry and a maximal $SU(2)^4$. As we will see, some of the models obtained in our scan can be interpreted as such asymmetric shift orbifolds involving $(-1)^{F_L}$. We denote this class as \mathcal{A}_d . Due to the appearance of $(-1)^{F_L}$, these models do not correspond to minima of any GSUGRA theory with only NS-NS fluxes.

What is also a common feature for these exactly solvable CFTs is that they correspond to special points with extra gauge enhancement. Going to the Higgs or Coulomb branch moves the model back to its generic locus. Working with type II models, such extra enhancements can only arise from the NS-NS sector. The gauge and matter fields appearing in the R-R sector are always abelian and uncharged.

3.4 ACFTs in $D = 8$

Let us now present the results of our scan over asymmetric simple current extensions in the $\mathbf{k} \in \{(1, 1, 1), (2, 2), (1, 4)\}$ Gepner models with $c = 3$. Due to the simplicity of these models we could check more than 10^8 simple current configurations but nevertheless we found only two different massless spectra.

The class ${}^8\mathfrak{N}_{[1,1]}$

The first class corresponds to the dimensional reduction of type IIA/B on \mathbb{T}^2 . This gives $\mathcal{N} = 2$ supersymmetry in eight dimensions with 32 supercharges and the ACFT data are

$${}^8\mathfrak{N}_{[1,1]} : \left\{ (1, 1, 1, 2)_L \otimes (1, 1, 1, 2)_R \quad \mathcal{G}_{(2)} \right\}. \quad (3.10)$$

Recall that the brackets count the number of states transforming in the (v, s, c, o) representation of the little group. The massless spectrum only consists of the $\mathcal{N} = 2$ supergravity multiplet with bosonic field content

$$\mathcal{G}_{(2)} = 1 \cdot [2] + 2 \cdot [\tfrac{3}{2}] + 6 \cdot [1] + 6 \cdot [\tfrac{1}{2}] + 7 \cdot [0] + 1 \cdot [t_3]. \quad (3.11)$$

For our notation and more details on the multiplet structure in eight dimensions see appendix A.1.

The class ${}^8\mathfrak{N}_{[0,1]}$

The second class, ${}^8\mathfrak{N}_{[0,1]}$, only appears for the $\mathbf{k} = (2, 2)$ model for instance with the additional asymmetric simply current

$$J = (0, 2, 2)(0, -2, 2)(v) \quad (3.12)$$

and has $\mathcal{N} = 1$ supersymmetry. The massless states arise in the ACFT as

$${}^8\mathfrak{N}_{[0,1]} : \left\{ (1, 0, 0, 6)_L \otimes (1, 1, 1, 2)_R \quad \mathcal{G}_{(1)} + 6 \cdot \mathcal{V}_{(1)} \right\}. \quad (3.13)$$

The spectrum fits into the supergravity multiplet with field content

$$\mathcal{G}_{(1)} = 1 \cdot [2] + 1 \cdot [\tfrac{3}{2}] + 2 \cdot [1] + 1 \cdot [\tfrac{1}{2}] + 1 \cdot [0] + 1 \cdot [t_2], \quad (3.14)$$

and six vectormultiplets with

$$\mathcal{V}_{(1)} = 1 \cdot [1] + 1 \cdot [\tfrac{1}{2}] + 2 \cdot [0]. \quad (3.15)$$

Note that there are no massless states from a left-moving Ramond sector. A closer look at these massless states reveals that these six vectors form the non-abelian gauge group $SU(2) \times SU(2)$. Recalling that the $\mathbf{k} = (2, 2)$ Gepner model corresponds to the rectangular \mathbb{T}^2 at self-dual radii $r_i = \sqrt{\alpha'}$ with $i = 1, 2$, this model is nothing else than the asymmetric $(-1)^{F_L}$ shift orbifold \mathcal{A}_8 discussed in section 3.3.

Summary

In Table 1 we summarize the ACFTs encountered in eight dimensions. Since

class	spectrum	realization
${}^8\mathfrak{N}_{[1,1]}$	$\mathcal{G}_{(2)}$	\mathbb{T}^2
${}^8\mathfrak{N}_{[0,1]}$	$\mathcal{G}_{(1)} + 6 \cdot \mathcal{V}_{(1)}^{SU(2)^2}$	\mathcal{A}_8

Table 1: Classification of type IIB ACFTs in 8D.

the \mathbb{T}^2 trivially does not have any three-cycles to support NS-NS fluxes, one does not expect to find any flux compactifications. This is consistent with our results where the only extra model is an orbifold that involves $(-1)^{F_L}$, which is an asymmetric operation of the Ramond sector.

However, the super-Higgs mechanism would in principle be possible. Indeed, using equation (A.2) from appendix A.1 we can decompose the eight-dimensional $\mathcal{N} = 2$ supergravity multiplet into $\mathcal{N} = 1$ multiplets as

$$\mathcal{G}_{(2)} \rightarrow \mathcal{G}_{(1)} + \overline{\mathcal{S}}_{(1)} + (2 - \alpha) \cdot \mathcal{V}_{(1)} + \alpha \cdot \overline{\mathcal{V}}_{(1)}, \quad (3.16)$$

with $\alpha = 0, 1, 2$. Hence, depending on the precise breaking mechanism, the spontaneously-broken $\mathcal{N} = 1$ theory can have 0, 1, 2 massless vector multiplets in addition to the supergravity multiplet. But, as mentioned before, since in string theory there are no suitable NS-NS fluxes available which would give rise to the breaking, we do not expect to find these models in our search.

3.5 ACFTs in $D = 6$

In this section we extend our investigation to the case $c = 6$, that corresponds to compactifications to six space-time dimensions. Using the framework described in section 2, we considered all Gepner models with $c = 6$ and added up to four additional simple currents. In general, these simple currents do not commute with some of the generic simple currents J_i and J_{GSO} . It is remarkable that within the millions of generated type IIB models, we found very few different massless spectra. The question therefore arises, whether these models correspond to a flux compactification on \mathbb{T}^4 or $K3$, respectively.

The class ${}^6\mathfrak{N}_{[2,2]}$

There exists the model with maximal $\mathcal{N} = (2, 2)$ supersymmetry, which is just the compactification of type II on a \mathbb{T}^4 . For instance the pure Gepner model $\mathbf{k} = (1, 1, 1, 2, 2)$ gives the massless spectrum

$${}^6\mathfrak{N}_{[2,2]} : \left\{ (1, 2, 2, 4)_L \otimes (1, 2, 2, 4)_R \quad \mathcal{G}_{(2,2)} \right\}. \quad (3.17)$$

All massless states fit into the $\mathcal{N} = (2, 2)$ supergravity multiplet, on which more details can be found in appendix A.2.

The classes ${}^6\mathfrak{N}_{[1,1]}$

Next, we consider the Gepner model $\mathbf{k} = (2, 2, 2, 2)$. The field content of the ACFT is

$${}^6\mathfrak{N}_{[1,1]}(\text{B}) : \left\{ \begin{array}{ll} (1, 0, 2, 0)_L \otimes (1, 0, 2, 0)_R & \mathcal{G}_{(0,2)} + \mathcal{T}_{(0,2)}, \\ 20 \times [(0, 0, 1, 2)_L \otimes (0, 0, 1, 2)_R] & 20 \cdot \mathcal{T}_{(0,2)}, \end{array} \right. \quad (3.18)$$

so that the model has 21 tensor-multiplets. Recall that the second row displays the number of massless states in the charged sector of the simple current extension. Geometrically this corresponds to a compactification of the type IIB superstring on a $K3$ -manifold.

The type IIA model can be realized by a simple current extension of the type IIB model. For $\mathbf{k} = (2, 2, 2, 2)$ adding the simple current

$$J_{\text{ACFT}} = (0, 1, 1)(0, 1, 1)(0, 1, 1)(0, 1, 1)(s) \quad (3.19)$$

gives an ACFT with

$${}^6\mathfrak{N}_{[1,1]}(\text{A}) : \begin{cases} (1, 2, 0, 0)_L \otimes (1, 0, 2, 0)_R & \mathcal{G}_{(1,1)} , \\ 20 \times [(0, 1, 0, 2)_L \otimes (0, 0, 1, 2)_R] & 20 \cdot \mathcal{V}_{(1,1)} . \end{cases} \quad (3.20)$$

Note the change of the $SO(4)$ representations $c \rightarrow s$ in the left-moving sector as compared to (3.18). Thus, we get a non-chiral $\mathcal{N} = (1, 1)$ supergravity theory with the massless spectrum of type IIA on $K3$. Recalling that IIA = IIB/ $(-1)^{F_L}$, the simple current J_{ACFT} is analogous to the quotient by $(-1)^{F_L}$.

The class ${}^6\mathfrak{N}_{[0,2]}$

Another model with 16 supercharges can be obtained by compactifying the asymmetric 8D model ${}^8\mathfrak{N}_{[0,1]}$ on a two-torus \mathbb{T}^2 . This can be realized as an ACFT by taking the Gepner model $\mathbf{k} = (1, 1, 1, 2, 2)$ and adding the simple current $J_{\text{ACFT}} = (0, 0, 0)^3(0, 2, 2)(0, -2, 2)(v)$ similar to (3.12). All states are coming from the vacuum sector with all left-moving Ramond states projected out

$${}^6\mathfrak{N}_{[0,2]} : \left\{ (1, 0, 0, 8)_L \otimes (1, 2, 2, 4)_R \quad \mathcal{G}_{(1,1)} + 8 \cdot \mathcal{V}_{(1,1)} . \right. \quad (3.21)$$

The eight vectors transform in the gauge group $SU(2) \times SU(2) \times U(1)^2$. Working with the $\mathbf{k} = (2, 2, 2, 2)$ Gepner model, we also found models with $n_V = 4, 8, 12$ vectors. Note that $n_V = 12$ is the maximal gauge enhancement for the model \mathcal{A}_6 with gauge group $SU(2)^4$ and $n_V = 4$ the minimal gauge group $U(1)^4$. Thus, we can consider all models found in this class as lying on the Coulomb-branch of \mathcal{A}_6 .

The class ${}^6\mathfrak{N}_{[0,1]}$

Now we come to the most interesting case, i.e. models for which the asymmetric simple current leads to minimal chiral $\mathcal{N} = (0, 1)$ supersymmetry. Remarkably, we only found one such class. One representative model is the $\mathbf{k} = (2, 2, 2, 2)$ Gepner model extended by one of the simple currents $J_{\text{ACFT}} = (0, 0, 0)^2(0, 1, 1)^2(v)$ or $J_{\text{ACFT}} = (0, 0, 0)^2(0, 2, 2)(0, -2, 2)(c)$. Both simple currents yield the same model and have a similar form as (3.45) or (3.12) that both implemented a $(-1)^{F_L}$ action. Beyond the vacuum orbit the model features three additional types of charged orbits

$${}^6\mathfrak{N}_{[0,1]} : \begin{cases} (1, 0, 0, 0)_L \otimes (1, 0, 2, 0)_R & \mathcal{G}_{(0,1)} + \mathcal{T}_{(0,1)} , \\ 8 \times [(0, 1, 0, 0)_L \otimes (0, 1, 0, 2)_R] & 8 \cdot \mathcal{T}_{(0,1)} , \\ 8 \times [(0, 0, 1, 0)_L \otimes (0, 1, 0, 2)_R] & 8 \cdot \mathcal{V}_{(0,1)} , \\ 20 \times [(0, 0, 0, 2)_L \otimes (0, 1, 0, 2)_R] & 20 \cdot \mathcal{H}_{(0,1)} . \end{cases} \quad (3.22)$$

Thus we have $\mathcal{N} = (0, 1)$ supersymmetry with the additional massless spectrum of tensor-, vector- and hypermultiplets

$$n_T = 1 + 8 = 9, \quad n_V = 8, \quad n_H = 20. \quad (3.23)$$

This spectrum satisfies the anomaly cancellation condition $n_H - n_V + 29n_T = 273$. From the ACFT analysis, we find an extension of this model with additional vectors and hypers, $n_V = 8 + n$ and $n_H = 20 + n$ with n up to four. The additional hypers arise from the charged sector similar as in (3.22), while the additional vectors come from scalars in a slightly modified vacuum sector

$$(1, 0, 0, n)_L \otimes (1, 0, 2, 0)_R \quad \mathcal{G}_{(0,1)} + \mathcal{T}_{(0,1)} + n \cdot \mathcal{V}_{(0,1)}. \quad (3.24)$$

For a simple example we checked that the CFT three-point functions $\langle v\psi\bar{\psi} \rangle$ satisfied all selection rules, indicating that the matter field ψ carries a non-trivial $U(1)$ charge under the abelian extra gauge fields. Therefore, these additional pairs of (vector + hyper) can be made massive via Higgsing.

Comments on ${}^6\mathfrak{N}_{[0,1]}$

Let us comment on the reason why the class of $\mathcal{N} = (0, 1)$ ACFTs in six dimensions is so restricted.

Anomalies First, we observe that anomaly cancellation in addition to some generic input from the ACFT construction allows to restrict $\mathcal{N} = (0, 1)$ ACFTs considerably. Say, we have found a type IIB model of this kind with massless spectrum $(n_{T,RR}^B + 1, n_{V,RR}^B + n_{V,NSNS}^B, n_{H,RR}^B)$ where we indicated the RR and NS-NS sector in the subscript.³ Then using the same ACFT, the corresponding type IIA model will have the spectrum

$$(n_{T,RR}^A + 1, n_{V,RR}^A + n_{V,NSNS}^A, n_{H,RR}^A) = (n_{V,RR}^B + 1, n_{T,RR}^B + n_{V,NSNS}^B, n_{H,RR}^B), \quad (3.25)$$

i.e. tensors and vectors from the R-R sector are exchanged while the states in the NS-NS sector match. States charged under R-R vectors do not exist, so that in 6D one only has the gravitational anomaly

$$\mathcal{A}_G = \alpha \text{Tr}(R^4) + \beta (\text{Tr } R^2)^2, \quad (3.26)$$

³Notice that the indicated structure of the multiplets is completely determined by the ACFT construction. The NS-NS multiplets can only come from the vacuum sector which, due to the $\mathcal{N} = (0, 1)$ worldsheet supersymmetry, is given by $(1, 0, 0, n) \otimes (1, 0, 2, 0)$ or its charge conjugate. There is therefore always exactly one tensor multiplet from the NS-NS sector and a so far not restricted amount of vector multiplets. All other multiplets must arise from the R-R sector.

where

$$\alpha \sim 244 - 29 n_T^{B/A} - n_H^{B/A} + n_V^{B/A}, \quad \beta \sim n_T^{B/A} - 8. \quad (3.27)$$

Requiring that both the type IIB and the correlated type IIA spectrum cancel the irreducible anomaly immediately leads to $n_{T,RR}^{B/A} = n_{V,RR}^{B/A}$. Moreover, for the type II superstring there are no Chern-Simons term so that, like for the ten-dimensional type IIB superstring, the reducible anomaly should also cancel right away. This leads to $n_{T,RR}^{B/A} = 8$ and following from that $n_{V,RR}^{B/A} = 8$ as well as $n_{H,RR} = 20 + n_{V,NSNS}$. To summarize, from this line of arguments one expects that the only consistent $\mathcal{N} = (0, 1)$ spectrum arising from our ACFT construction is the one we found.

Fluxes From a supergravity point of view, a supersymmetry breaking to $\mathcal{N} = (0, 1)$ could in principle be achieved by turning on fluxes on the $K3$. However, the ACFT is expected to only contain NS-NS fluxes, which carry three indices. Since the $K3$ only contains two-cycles, the usual geometric and non-geometric fluxes H_{ijk} , $F_{ij}{}^k$, $Q_i{}^{jk}$, R^{ijk} cannot be supported. Thus, the class ${}^6\mathfrak{N}_{[0,1]}$ cannot be considered as an NS-NS flux compactification of $K3$.

In agreement with this observation, we mention that an $\mathcal{N} = (0, 1)$ model in six dimensions can not be obtained via a spontaneous susy-breaking mechanism. For a spontaneously-broken $(0, 1)$ -vacuum, gravitinos have to become massive and have to be part of a massive spin-3/2 multiplet. When decomposing for instance the $(2, 2)$ -theory in six dimensions into $(0, 1)$ -multiplets using the relations in (A.4), we find

$$\mathcal{G}_{(2,2)} \rightarrow \mathcal{G}_{(0,1)} + 4 \cdot \mathcal{S}_{(0,1)}^+ + 2 \cdot \mathcal{S}_{(0,1)}^- + 8 \cdot \mathcal{V}_{(0,1)} + 5 \cdot \mathcal{T}_{(0,1)} + 10 \cdot \mathcal{H}_{(0,1)}. \quad (3.28)$$

However, a massive spin-3/2 multiplets contains one chiral and one anti-chiral gravitino, and hence not all gravitinos in (3.28) can become massive. Thus, spontaneous susy-breaking from $\mathcal{N} = (2, 2)$ to $\mathcal{N} = (0, 1)$ in six dimensions is not possible. For the breaking from $\mathcal{N} = (1, 1)$ or $\mathcal{N} = (0, 2)$ to $\mathcal{N} = (0, 1)$ a similar reasoning applies. Using the relations in (A.4) we find for theories with n_T tensor or n_V vector multiplets

$$\begin{aligned} \mathcal{G}_{(0,2)} + n_T \cdot \mathcal{T}_{(0,2)} &\rightarrow \mathcal{G}_{(0,1)} + 2 \cdot \mathcal{S}_{(0,1)}^- + n_T \cdot \mathcal{T}_{(0,1)} + 2n_T \cdot \mathcal{H}_{(0,1)}, \\ \mathcal{G}_{(1,1)} + n_V \cdot \mathcal{V}_{(1,1)} &\rightarrow \mathcal{G}_{(0,1)} + 2 \cdot \mathcal{S}_{(0,1)}^+ + \mathcal{T}_{(0,1)} + n_V \cdot \mathcal{V}_{(0,1)} + 2n_V \cdot \mathcal{H}_{(0,1)}. \end{aligned} \quad (3.29)$$

Again, the gravitinos cannot become massive and hence spontaneous supersymmetry-breaking is not possible.

Orbifold realization The above-mentioned model was discussed before in the literature [10], where also the following toroidal orbifold realization was provided

$$\text{Model 6D} = \frac{\mathbb{T}^4}{\mathbb{Z}_2 \times \mathbb{Z}_2'}. \quad (3.30)$$

With the reflection

$$\Theta : z_i \rightarrow -z_i, \quad i = 1, 2, \quad (3.31)$$

the two discrete symmetries are given by Θ and $\Theta S(-1)^{F_L}$. Here S denotes the momentum shift operator along a single circle S^1 . Note that as expected from the form of the simple currents also this orbifold involves the asymmetric operation $(-1)^{F_L}$ that is only defined on the Ramond sector.

Summary

To summarize, the type IIB ACFT models with $c = 6$ obtained in our scan are rather restricted and can be characterized as shown in Table 2. From this table

class	spectrum beyond SUGRA	realization
${}^6\mathfrak{N}_{[2,2]}$	—	${}^8\mathfrak{N}_{[1,1]}$ on \mathbb{T}^2
${}^6\mathfrak{N}_{[1,1]}(\text{B})$	$21 \cdot \mathcal{T}_{(0,2)}$	IIB on $K3$
${}^6\mathfrak{N}_{[1,1]}(\text{A})$	$20 \cdot \mathcal{V}_{(1,1)}$	IIB on $K3/(-1)^{F_L} = \text{IIA on } K3$
${}^6\mathfrak{N}_{[0,2]}$	$(4, 8, 12) \cdot \mathcal{V}_{(1,1)}$	Coulomb-branch: \mathcal{A}_6
${}^6\mathfrak{N}_{[0,1]}$	$9 \cdot \mathcal{T}_{(0,1)} + (8 + n) \cdot \mathcal{V}_{(0,1)}$ $+ (20 + n) \cdot \mathcal{H}_{(0,1)}$	gauge enhancement: $\mathbb{T}^4/\{\Theta, \Theta S(-1)^{F_L}\}$

Table 2: Classification of type IIB ACFTs in 6D.

it is clear that there is indeed no room for genuine NS-NS flux compactifications. All asymmetric models are realized by orbifolds involving the projection $(-1)^{F_L}$, that only exists for the superstring and which is of course related to the existence of a Ramond sector. Moreover, consistent with [45] we did not find any model with $\mathcal{N} = (1, 2)$ supersymmetry.

3.6 ACFTs in $D = 4$

In this section we consider type II Gepner models with $c = 9$ and initial $\mathcal{N} = 4$ supersymmetry, corresponding to compactifications on $K3 \times \mathbb{T}^2$. Again we constructed of the order to 10^8 individual models⁴, whose massless spectra however fit into a small number of different classes. For most of them, we can find a representative starting with the $\mathbf{k} = (1^3, 2^4)$ Gepner model. As will be presented in this section, there are models with $\mathcal{N} = 8, 6, 5, 4, 3, 2$ supersymmetry. Similarly to six dimensions, some of the resulting models can be understood as toroidal

⁴To give some numbers: Only in the $k = (2^6)$ model the stochastic search included 4.3 million different ACFTs.

compactifications of models in higher dimensions. However, we will also consider the possibility that some of the classes are flux compactifications on \mathbb{T}^6 or $K3 \times \mathbb{T}^2$. Note that the latter manifold also contains three-cycles that can support NS-NS fluxes. We will derive necessary constraints for such GSUGRA models and compare them with the ACFT results.

Let us present the classes found in the order of decreasing number of supersymmetries. Recall that our initial model, like $\mathbf{k} = (1^3, 2^4)$, has at least two right-moving supersymmetries so that compactification on genuine Calabi-Yau three-folds are not in our class.

The classes ${}^4\mathfrak{N}_{[4,4]}$, ${}^4\mathfrak{N}_{[2,4]}$ and ${}^4\mathfrak{N}_{[1,4]}$

We begin by extending the $\mathbf{k} = (1^3, 2^4)$ Gepner model by the simple current

$$J_{\text{ACFT}} = (0, 0, 0)^2(0, 1, 1); (0, 4, 0)^2(0, 3, -1)^2(o), \quad (3.32)$$

which leads to an extended $\mathcal{N} = 8$ supersymmetry. All the massless states arise in the vacuum orbit

$${}^4\mathfrak{N}_{[4,4]} : \left\{ (1, 4, 4, 6)_L \otimes (1, 4, 4, 6)_R \right\} \quad \mathcal{G}_{(8)}. \quad (3.33)$$

Clearly, this model corresponds to type IIB string theory compactified on a \mathbb{T}^6 .

Extending instead by the simple current

$$J_{\text{ACFT}} = (0, -2, 0)(0, 3, -1)(0, -2, 2)(0, 1, 1)^2(0, 2, 2)^2(v), \quad (3.34)$$

leads to a model with $\mathcal{N} = 6$ supersymmetry

$${}^4\mathfrak{N}_{[2,4]} : \left\{ (1, 2, 2, 2)_L \otimes (1, 4, 4, 6)_R \right\} \quad \mathcal{G}_{(6)}. \quad (3.35)$$

Kinematically, this model can be interpreted as a partial supersymmetry breaking by fluxes. Indeed, as discussed in section 3.2, the massless supergravity multiplet $\mathcal{G}_{(8)}$ splits into the massless supergravity multiplet $\mathcal{G}_{(6)}$ plus a massive spin-3/2 multiplet. We note that the $\mathcal{N} = 6$ model also admits a toroidal orbifold realization [4] as $\mathbb{T}^6/(\mathbb{Z}_2^L S)$, where \mathbb{Z}_2^L denotes a purely left-moving reflection of four compact coordinates and S is a \mathbb{Z}_2 shift along the orthogonal \mathbb{T}^2 . Note that this orbifold does not contain a factor $(-1)^{F_L}$, that would transcend a pure NS-NS flux realization.

A further breaking of the above model to $\mathcal{N} = 5$ supersymmetry can be achieved by the extension with two simple currents

$$\begin{aligned} J_{\text{ACFT},1} &= (0, -1, 1)(0, 3, -1)(0, 2, 0)(0, 4, 0)(0, 2, 2)(0, 1, 1)(0, 3, -1)(v), \\ J_{\text{ACFT},2} &= (0, -1, 1)(0, 2, 2)(0, 2, 0)(0, -1, -1)(0, 2, 2)(0, 1, 1)(0, 2, 2)(o), \end{aligned} \quad (3.36)$$

yielding

$${}^4\mathfrak{N}_{[1,4]} : \left\{ (1, 1, 1, 0)_L \otimes (1, 4, 4, 6)_R \quad \mathcal{G}_{(5)} \right\}. \quad (3.37)$$

Note that for this model the maximal $\mathcal{G}_{(8)}$ multiplet does not split into the massless supergravity multiplet $\mathcal{G}_{(5)}$ plus a number of massive spin-3/2 multiplets. Thus, there is no super Higgs effect at work. However, there exist an asymmetric orbifold realization, $\mathbb{T}^6/(\mathbb{Z}_2^L S, \tilde{\mathbb{Z}}_2^L \tilde{S})$, with two orthogonal shifted asymmetric reflections.

The class ${}^4\mathfrak{N}_{[0,4]}$

Next we come to models featuring $\mathcal{N} = 4$ supersymmetry in 4D. First there exists a class that can be considered as the continuation of the models $\mathcal{N} \geq 5$ just described. The massless spectrum arises completely from the left-moving NS-sector as

$${}^4\mathfrak{N}_{[0,4]} : \left\{ (1, 0, 0, n)_L \otimes (1, 4, 4, 6)_R \quad \mathcal{G}_{(4)} + n \cdot \mathcal{V}_{(4)} \right\}. \quad (3.38)$$

We found a series of models with $n_V = 0, 2, 4, 6, 8, 10, 14, 18$. The model with $n_V = 18$ can be considered as the model \mathcal{A}_4 with maximal non-abelian gauge symmetry $SU(2)^6$. Going to the Coulomb-branch can give the models with $n_V \geq 6$. The model with $n_V = 6$ vector multiplets also arises from the super Higgs effect

$$\mathcal{G}_{(8)} \rightarrow \mathcal{G}_{(4)} + 6 \cdot \mathcal{V}_{(4)} \quad (3.39)$$

plus two massive 1/2-BPS gravitino supermultiplets. Since one massive vector-multiplet consists of two massless ones, the models with $n_V = 4, 2, 0$ can also be explained by flux compactifications on \mathbb{T}^6 . Thus, the class ${}^4\mathfrak{N}_{[0,4]}$ can be fully explained by two different mechanisms.

The class ${}^4\mathfrak{N}_{[2,2]}$

Let us now turn to the class ${}^4\mathfrak{N}_{[2,2]}$. Their massless spectrum arises from both the vacuum and extra matter orbits as

$${}^4\mathfrak{N}_{[2,2]} : \left\{ \begin{array}{ll} (1, 2, 2, 2)_L \otimes (1, 2, 2, 2)_R & \mathcal{G}_{(4)} + 2 \cdot \mathcal{V}_{(4)}, \\ n \times [(0, 1, 1, 2)_L \otimes (0, 1, 1, 2)_R] & n \cdot \mathcal{V}_{(4)}. \end{array} \right. \quad (3.40)$$

Concretely, we obtained $n_V = 22, 14, 10, 6, 4$. The first model is just the $K3 \times \mathbb{T}^2$ compactification of type IIB.

The question arises whether one can find an interpretation of the other four models with $n_V \in \{4, 6, 10, 14\}$. Since the massless spectrum is not asymmetric,

one might suspect that there exist corresponding geometric compactifications such as toroidal orbifolds. Indeed such toroidal orbifolds with $\mathcal{N} = 4$ are given by

$$\text{Orb}_{n,m} = \frac{\mathbb{T}^4 \times \mathbb{T}^2}{\mathbb{Z}_n S_m}, \quad (3.41)$$

where \mathbb{T}^4 is chosen such that it admits a crystallographic action of \mathbb{Z}_n for $n \in \{2, 3, 4, 6\}$. Moreover, S_m denotes a momentum shift of order m on \mathbb{T}^2 , where m is required to be a divisor of n . The computation of the resulting massless spectra is straightforward and listed in table 3. Thus, the orbifolds provide precisely the numbers found in the ACFT construction.

$\text{Orb}_{n,m}$	twisted sector vectors	massless spectrum
$(2, 2)$	$(1, \theta) = (6, 0)$	$\mathcal{G}_{(4)} + 6 \cdot \mathcal{V}_{(4)}$
$(3, 3)$	$(1, \theta, \theta^2) = (4, 0, 0)$	$\mathcal{G}_{(4)} + 4 \cdot \mathcal{V}_{(4)}$
$(4, 2)$	$(1, \theta, \theta^2, \theta^3) = (4, 0, 10, 0)$	$\mathcal{G}_{(4)} + 14 \cdot \mathcal{V}_{(4)}$
$(6, 3)$	$(1, \theta, \theta^2, \theta^3, \theta^4, \theta^5) = (4, 0, 0, 6, 0, 0)$	$\mathcal{G}_{(4)} + 10 \cdot \mathcal{V}_{(4)}$

Table 3: Spectra of shift orbifolds. The spectra of the other models $\text{Orb}_{4,4}$, $\text{Orb}_{6,6}$ and $\text{Orb}_{6,2}$ give $n_V = \{4, 4, 14\}$, which are already contained in the list.

The class ${}^4\mathfrak{N}_{[1,2]}$

We also found a class of models featuring $\mathcal{N} = 3$ supersymmetry in 4D. One representative originates from the $\mathbf{k} = (1^3, 2^4)$ Gepner model via extension by the simple current

$$J_{\text{ACFT}} = (0, 3, -1)(0, 1, 1)(0, -2, 0)(0, -1, -1)(0, -2, 2)(0, 4, 0)(0, -3, 1)(s). \quad (3.42)$$

The massless spectrum reads

$${}^4\mathfrak{N}_{[1,2]} : \begin{cases} (1, 1, 1, 0)_L \otimes (1, 2, 2, 2)_R & \mathcal{G}_{(3)} + \mathcal{V}_{(3)}, \\ 6 \times [(0, 2, 0, 2)_L \otimes (0, 1, 1, 2)_R] & 6 \cdot \mathcal{V}_{(3)}, \\ 6 \times [(0, 0, 2, 2)_L \otimes (0, 1, 1, 2)_R] & 6 \cdot \mathcal{V}_{(3)}. \end{cases} \quad (3.43)$$

Thus, besides the $\mathcal{N} = 3$ supergravity multiplet there are 13 vectormultiplets. Our stochastic search also provided models with

$$n_V \in \{3, 7, 11, 13, 19\}. \quad (3.44)$$

Let us analyze whether this class could arise via the super Higgs effect from a 4D theory with higher supersymmetry, which would for instance correspond to a flux compactification on \mathbb{T}^4 or $K3 \times \mathbb{T}^2$. Using the supermultiplet structure reviewed in appendix A.3, one can straightforwardly derive the table 4 for admissible super Higgsing. Therefore, all the models we found with $n_V \leq 19$ can

\mathcal{N}'	\mathcal{N}	massless spectrum
8	3	$\mathcal{G}_3 + (3 - 2k) \cdot \mathcal{V}_3$
6	3	—
5	3	—
4	3	$\mathcal{G}_3 + (19 - 2k) \cdot \mathcal{V}_3$

Table 4: Admissible super Higgs effect $\mathcal{N}' \rightarrow \mathcal{N} = 3$ with $k \in \mathbb{N}_0$.

be understood as flux compactifications on $K3 \times \mathbb{T}^2$. The model with $n_V = 3$ could also be interpreted as arising via super Higgs mechanism from $\mathcal{N}' = 8$, i.e. a flux model on \mathbb{T}^6 . Note that the ACFT data are completely consistent with an interpretation in terms of a super Higgs effect, i.e. a Minkowski type flux vacuum. We see the upper bound $n_V = 19$ and only odd numbers of vector multiplets.

The class ${}^4\mathfrak{N}_{[0,2]}$

We now turn to the case of $\mathcal{N} = 2$ supersymmetry in 4D. Of course, most Gepner models directly give models of the type ${}^4\mathfrak{N}_{[1,1]}$, corresponding to compactifications of type IIB string theory on genuine Calabi-Yau three-folds. However, here we are not interested in these cases. Instead we search for ACFTs in the class ${}^4\mathfrak{N}_{[0,2]}$, as these could arise from Minkowski minima of flux compactifications on $K3 \times \mathbb{T}^2$.

We find only three different classes of models that are distinguished by the difference of the number of vector and hypermultiplets. The massless spectrum of the first class reads

$${}^4\mathfrak{N}_{[0,2]}(\mathbf{A}) : \begin{cases} (1, 0, 0, m)_L \otimes (1, 2, 2, 2)_R & \mathcal{G}_{(2)} + (m + 1) \cdot \mathcal{V}_{(2)} , \\ (0, n, n, 2k)_L \otimes (0, 1, 1, 2)_R & 2n \cdot \mathcal{V}_{(2)} + k \cdot \mathcal{H}_{(2)} , \end{cases} \quad (3.45)$$

with $n \geq 1$. Thus, such models have $n_V = m + 2n + 1$ vectormultiplets and $n_H = k$ hypermultiplets. In our search we always find $n_H = n_V + 1$ with a long list for the number of vectormultiplets

$$n_V \in \{1, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 20, 21, 22, 23\} . \quad (3.46)$$

- The model with $n_V = 19$ can be interpreted as the 6D model ${}^6\mathfrak{N}_{[0,1]}$ compactified on \mathbb{T}^2 . As in 6D, we find up to four additional vector/hyper pairs.

- Let us analyze the possibility of the super Higgs effect to occur for $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2$ in four dimensions. Let us denote by $n_{3/2}^{L,S}$ the number of long and short massive $\mathcal{N} = 2$ gravitino multiplets and by $n_1^{L,S}$ the number of long and short massive $\mathcal{N} = 2$ vector multiplets. The super Higgs effects leave

$$\begin{aligned} n_V &= 27 - 4(n_{3/2}^L + n_{3/2}^S) - (n_1^L + 2n_1^S) \quad \text{and} \\ n_H &= 20 + 4n_{3/2}^S - n_1^L \end{aligned} \quad (3.47)$$

massless vector- and hypermultiplets. For the case of no massive short multiplets, one finds $n_V = 19 - n_1^L$ and $n_H = 20 - n_1^L$ so that indeed $n_H - n_V = 1$. Therefore, *all* the models with $n_V \leq 19$ are consistent with the expectation from a super Higgs mechanism. We find it quite remarkable that our list of ACFTs covers (almost) all possible values of n_V . We expect that the few gaps will also be filled by running an even more extensive search.

Let us emphasize that precisely for the case of flux compactifications on $K3 \times \mathbb{T}^2$, we find an increase in the number of ACFTs, all of them consistent with the GSUGRA predictions $n_H - n_V = 1$ and $n_V \leq 19$.

The second class ${}^4\mathfrak{N}_{[0,2]}(\text{B})$ has a massless spectrum of the same form (3.45) though obeys $n_V - n_H = 11$ with

$$n_V \in \{13, 15, 17, 19, 21, 23\}. \quad (3.48)$$

Before interpreting the second class let us introduce the third class consisting of models without massless states from the left-moving Ramond sector. The massless spectrum has the following structure

$${}^4\mathfrak{N}_{[0,2]}(\text{C}) : \begin{cases} (1, 0, 0, m)_L \otimes (1, 2, 2, 2)_R & \mathcal{G}_{(2)} + (m+1) \cdot \mathcal{V}_{(2)}, \\ (0, 0, 0, 2k)_L \otimes (0, 1, 1, 2)_R & k \cdot \mathcal{H}_{(2)}. \end{cases} \quad (3.49)$$

All models obtained in our scan corresponding to this class satisfy $n_H - n_V = 13$ with

$$n_V \in \{3, 4, 5, 7, 8, 9, 10, 11\}. \quad (3.50)$$

The even models were very rare so that we are confident that a more-extensive scan would fill the gaps.

- For $n_V \leq 7$ the models can arise via the super Higgs effect from $\mathcal{N} = 4$ through $n_{3/2}^S = 0$ and $n_1^S = 6$. In this case, we find $n_V = 7 - n_1^L$ and $n_H = 20 - n_1^L$.

- Alternatively, the model with $n_V = 7$ also results from compactifying the asymmetric 8D model ${}^8\mathfrak{N}_{[0,1]} = \mathcal{A}_8$ on a $K3$ manifold. One way to see this is to consider the orbifold realization

$$\frac{\mathbb{T}^4 \times \mathbb{T}^2}{\{\mathbb{Z}_2, (-1)^{F_L} SW\}}, \quad (3.51)$$

where the \mathbb{Z}_2 acts as a reflection $\theta : x_i \rightarrow -x_i$ on the four coordinates of \mathbb{T}^4 . Having the structure of a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, one can introduce a discrete torsion $\epsilon = \pm 1$. In Table 5 we display the resulting massless spectra for these two choices, indicating the various twisted sector contributions. As one can see the choice of $\epsilon = 1$ gives a spectrum from the third class while $\epsilon = -1$ fits into the second class of ACFT models.

Note that for the $\epsilon = +1$ model there are no massless modes from the R-R sector, both in the untwisted and twisted sectors. On the other hand, for the $\epsilon = -1$ model in the θ -twisted sector the NS-NS sector is projected out and the R-R sector is kept.

Therefore the last two ACFT classes above can be interpreted as the Higgs branches of these two models. We note that, as observed before in 6D, up to four additional vector/hyper pairs can become massless. In the moment, we cannot say whether this is a strong upper bound that maybe has a natural interpretation.

sector	$\epsilon = +1$	$\epsilon = -1$
untwisted	$\mathcal{G}_{(2)} + 3 \cdot \mathcal{V}_{(2)} + 4 \cdot \mathcal{H}_{(2)}$	$\mathcal{G}_{(2)} + 3 \cdot \mathcal{V}_{(2)} + 4 \cdot \mathcal{H}_{(2)}$
θ twisted	$16 \cdot \mathcal{H}_{(2)}$	$16 \cdot \mathcal{V}_{(2)}$
$(-1)^{F_L} SW$ twisted	$4 \cdot \mathcal{V}_{(2)}$	$4 \cdot \mathcal{H}_{(2)}$
total	$\mathcal{G}_{(2)} + 7 \cdot \mathcal{V}_{(2)} + 20 \cdot \mathcal{H}_{(2)}$	$\mathcal{G}_{(2)} + 19 \cdot \mathcal{V}_{(2)} + 8 \cdot \mathcal{H}_{(2)}$

Table 5: Massless spectra for the orbifold (3.51) with and without discrete torsion.

Summary

To summarize, the four-dimensional type IIB ACFT models with $c = 9$ are rather restricted. In particular, they can be characterized according to the classification shown in Table 6.

4 Conclusions

In this paper we have presented the results of an extensive stochastic computer search for simple current extended asymmetric Gepner models with at least eight

class	spectrum beyond SUGRA	realization
${}^4\mathfrak{N}_{[4,4]}$	—	type IIB on \mathbb{T}^6
${}^4\mathfrak{N}_{[2,4]}$	—	sHiggs of ${}^4\mathfrak{N}_{[4,4]}$
${}^4\mathfrak{N}_{[1,4]}$	—	—
${}^4\mathfrak{N}_{[0,4]}$	$(0, 2, 4, 6) \cdot \mathcal{V}_{(4)}$ $(6, 8, 10, 14, 18) \cdot \mathcal{V}_{(4)}$	sHiggs of ${}^4\mathfrak{N}_{[4,4]}$ Coulomb branch: \mathcal{A}_4
${}^4\mathfrak{N}_{[2,2]}(\text{A})$	$22 \cdot \mathcal{V}_{(4)}$	type IIB on $K3 \times \mathbb{T}^2$
${}^4\mathfrak{N}_{[2,2]}(\text{B})$	$(4, 6, 10, 14) \cdot \mathcal{V}_{(4)}$	shift orbifolds $\text{Orb}_{n,m}$
${}^4\mathfrak{N}_{[1,2]}$	$(3, 7, 11, 13, 19) \cdot \mathcal{V}_{(3)}$	sHiggs of ${}^4\mathfrak{N}_{[2,2]}(\text{A})$
${}^4\mathfrak{N}_{[0,2]}(\text{A})$	$(1, \dots, 19) \cdot \mathcal{V}_{(2)} + (2, \dots, 20) \cdot \mathcal{H}_{(2)}$ $(19, \dots, 23) \cdot \mathcal{V}_{(2)} + (20, \dots, 24) \cdot \mathcal{H}_{(2)}$	sHiggs of ${}^4\mathfrak{N}_{[2,2]}(\text{A})$ ${}^6\mathfrak{N}_{[0,1]}$ on \mathbb{T}^2
${}^4\mathfrak{N}_{[0,2]}(\text{B})$	$(13, 15, 17, 19) \cdot \mathcal{V}_{(2)} + (2, 4, 6, 8) \cdot \mathcal{H}_{(2)}$ $(21, 23) \cdot \mathcal{V}_{(2)} + (10, 12) \cdot \mathcal{H}_{(2)}$	Higgs chain: ${}^8\mathfrak{N}_{[0,1]}$ on $K3_{\epsilon=-1}$ gauge enhancement
${}^4\mathfrak{N}_{[0,2]}(\text{C})$	$(3, 4, 5, 7) \cdot \mathcal{V}_{(2)} + (16, 17, 18, 20) \cdot \mathcal{H}_{(2)}$ $(8, 9, 10, 11) \cdot \mathcal{V}_{(2)} + (21, 22, 23, 24) \cdot \mathcal{H}_{(2)}$	Higgs chain: ${}^8\mathfrak{N}_{[0,1]}$ on $K3_{\epsilon=+1}$ gauge enhancement

Table 6: Classification of type IIB ACFTs in 4D.

supercharges in the right-moving sector. Our main motivation was to continue the analysis of four-dimensional $\mathcal{N} = 1$ ACFTs from [16] in a simpler setting, where a clearer picture could arise. The main result of [16] was a proposal for the identification of certain ACFTs as Minkowski minima of gauged supergravities in specified Calabi-Yau three-folds. This proposal had some ambiguities related to the existence of a scalar potential in four-dimensional $\mathcal{N} = 1$ theories. On the other hand, for models with eight supercharges considered in this paper, the generation of mass terms and scalar potentials is much more restricted, as often gauge fields and scalars reside in the same supermultiplet.

We considered ACFTs in $D = 8, 6, 4$ dimensions and were able to characterize our results by just a few classes. In 8D we only found two models, where one was just the \mathbb{T}^2 compactification of type IIB string theory in 10D. The second model could be identified with an asymmetric orbifold that involved momentum/winding shifts and $(-1)^{F_L}$. Moreover, rather reminiscent of the heterotic string, there appeared a non-abelian gauge symmetry. This model makes it very clear that there exist asymmetric ACFTs that cannot correspond to NS-NS flux

compactifications, as they involve the Ramond sector.

In 6D, the number of different ACFTs only increased slightly, all of them again being describable via asymmetric orbifolds involving $(-1)^{F_L}$. As we argued, there was no sign of the super Higgs mechanism, a prerequisite of spontaneous supersymmetry breaking via fluxes or gaugings, respectively. And indeed, since there are no three-cycles on $K3$, NS-NS fluxes cannot be supported and hence spontaneous partial susy breaking is not expected.

In 4D the landscape became richer but still the models could be classified according to their supersymmetry and the four categories: dimensional reduction, asymmetric $(-1)^{F_L}$ shift orbifolds, special CFT gauge enhancement and, last but not least, the super Higgs mechanism. The latter was expected via flux compactifications on \mathbb{T}^6 and $K3 \times \mathbb{T}^2$ and indeed for $\mathcal{N} = 3$ and $\mathcal{N} = 2$ supersymmetry two chains of models were obtained that precisely fit into this scheme. Of course, this does not yet prove that $\mathcal{N} = 4$ gauged supergravity really admits these Minkowski vacua by concrete choices of gaugings, but it provides compelling evidence. In fact, at least for certain models there could exist also an asymmetric orbifold realization (not involving $(-1)^{F_L}$), though this does not exclude an interpretation in terms GSUGRA. As in the free fermion construction [4], we found models with $\mathcal{N} = 8, 6, 5, 4, 3, 2$ supersymmetry. Of course, in view of the “landscape versus swampland” question, it would be desirable to identify the precise relation between the ACFT data and the concrete gaugings or fluxes. This is not an easy question and is beyond the scope of this paper.

To summarize, the landscape of asymmetric Gepner models is rich but still features a clear structure. Of course, these models do not cover all parts of the string landscape. In particular, we do not expect to find models with R-R fluxes turned on. Moreover, probably not all modular invariant partition functions can be reached via the simple current construction. There could well be string islands [41] that only a full classification of modular invariant partition functions can reveal.

Acknowledgments: We would like to thank O. Andreev, G. Dall’Agata and I. García-Etxebarria for helpful discussions.

A Supermultiplets

In this appendix we review the field content of the supermultiplets in various dimensions. Part of this information can be found in [46], and the full summary for four dimensions in [40].

A.1 Supergravity in $D = 8$

In this section we collect some information about the multiplet structure in eight dimensions. The on-shell degrees of freedom of the various fields are summarized as follows

name	symbol	on-shell d.o.f.	
massless spin 2	$[2]$	20_{b}	
massless spin 3/2	$[\frac{3}{2}]$	40_{f}	
massless spin 1	$[1]$	6_{b}	
massless spin 1/2	$[\frac{1}{2}]$	8_{f}	
massless spin 0	$[0]$	1_{b}	
massless p -form	$[t_p]$	$\binom{6}{p}_{\text{b}}$	(A.1)
massive spin 3/2	$\overline{[\frac{3}{2}]}$	48_{f}	
massive spin 1	$\overline{[1]}$	7_{b}	
massive spin 1/2	$\overline{[\frac{1}{2}]}$	8_{f}	
massive spin 0	$\overline{[0]}$	1_{b}	
massive p -form	$\overline{[t_p]}$	$\binom{7}{p}_{\text{b}}$	

The multiplets relevant for our discussion are summarized in table 7. In terms of the field content, these multiplets satisfy the following relations

$$\begin{aligned}
\mathcal{G}_{(2)} &= \mathcal{G}_{(1)} + \mathcal{S}_{(1)} + 2 \cdot \mathcal{V}_{(1)} , \\
\overline{\mathcal{S}}_{(1)} &= \mathcal{S}_{(1)} , \\
\overline{\mathcal{V}}_{(1)} &= \mathcal{V}_{(1)} .
\end{aligned}
\tag{A.2}$$

\mathcal{N}	spin	mass	content
2	2		$\mathcal{G}_{(2)} = 1 \cdot [2] + 2 \cdot [\frac{3}{2}] + 6 \cdot [1] + 6 \cdot [\frac{1}{2}] + 7 \cdot [0] + 1 \cdot [t_3] + 3 \cdot [t_2]$
1	2		$\mathcal{G}_{(1)} = 1 \cdot [2] + 1 \cdot [\frac{3}{2}] + 2 \cdot [1] + 1 \cdot [\frac{1}{2}] + 1 \cdot [0] + 1 \cdot [t_2]$
1	3/2		$\mathcal{S}_{(1)} = 1 \cdot [\frac{3}{2}] + 2 \cdot [1] + 3 \cdot [\frac{1}{2}] + 2 \cdot [0] + 2 \cdot [t_2] + 1 \cdot [t_3]$
1	3/2	long	$\overline{\mathcal{S}}_{(1)} = 1 \cdot \overline{[\frac{3}{2}]} + 1 \cdot \overline{[1]} + 2 \cdot \overline{[\frac{1}{2}]} + 1 \cdot \overline{[0]} + 1 \cdot \overline{[t_2]} + 1 \cdot \overline{[t_3]}$
1	1		$\mathcal{V}_{(1)} = 1 \cdot [1] + 1 \cdot [\frac{1}{2}] + 2 \cdot [0]$
1	1	long	$\overline{\mathcal{V}}_{(1)} = 1 \cdot \overline{[1]} + 1 \cdot \overline{[\frac{1}{2}]} + 1 \cdot \overline{[0]}$

Table 7: Supergravity multiplets in $D = 8$. The first column shows the amount of supersymmetry, the second column indicates the maximal spin of the multiplet, the third column specifies whether the fields are massless (no indication) or massive (long).

A.2 Supergravity in $D = 6$

In six dimensions, the on-shell degrees of freedom of the various fields are summarized as follows

name	symbol	on-shell d.o.f.
massless spin 2	$[2]$	9_b
massless spin 3/2	$[\frac{3}{2}]^\pm$	6_f
massless spin 1	$[1]$	4_b
massless spin 1/2	$[\frac{1}{2}]^\pm$	2_f
massless spin 0	$[0]$	1_b
massless two-form	$[t_2]^\pm$	3_b
massive spin 3/2	$\overline{[\frac{3}{2}]}$	16_f
massive spin 1	$\overline{[1]}$	5_b
massive spin 1/2	$\overline{[\frac{1}{2}]}$	4_f
massive spin 0	$\overline{[0]}$	1_b
massive two-form	$\overline{[t_2]}$	10_b

(A.3)

Note that the \pm indicates the chirality of the fermionic fields, and whether the two-tensor is self- or anti-self-dual. The multiplets relevant for our discussion are summarized in table 8. At the level of the field content, the following relations can be obtained

$$\begin{aligned}
\mathcal{G}_{(2,2)} &= \mathcal{G}_{(0,2)} + 4 \cdot \mathcal{S}_{(0,2)} + 5 \cdot \mathcal{T}_{(0,2)} , \\
\mathcal{G}_{(2,2)} &= \mathcal{G}_{(1,1)} + 2 \cdot \mathcal{S}_{(1,1)}^+ + 2 \cdot \mathcal{S}_{(1,1)}^- + 4 \cdot \mathcal{V}_{(1,1)} , \\
\mathcal{G}_{(0,2)} &= \mathcal{G}_{(0,1)} + 2 \cdot \mathcal{S}_{(0,1)}^- , & \mathcal{G}_{(1,1)} &= \mathcal{G}_{(0,1)} + 2 \cdot \mathcal{S}_{(0,1)}^+ + \mathcal{T}_{(0,1)} , \\
\mathcal{S}_{(0,2)} &= \mathcal{S}_{(0,1)}^+ + 2 \cdot \mathcal{V}_{(0,1)} , & \mathcal{S}_{(1,1)}^+ &= \mathcal{S}_{(0,1)}^+ + 2 \cdot \mathcal{T}_{(0,1)} , \\
\mathcal{T}_{(0,2)} &= \mathcal{T}_{(0,1)} + 2 \cdot \mathcal{H}_{(0,1)} , & \mathcal{S}_{(1,1)}^- &= \mathcal{S}_{(0,1)}^- + 2 \cdot \mathcal{V}_{(0,1)} + \mathcal{H}_{(0,1)} , \\
& & \mathcal{V}_{(1,1)} &= \mathcal{V}_{(0,1)} + 2 \cdot \mathcal{H}_{(0,1)} , \\
\overline{\mathcal{S}}_{(2)} &= \mathcal{S}_{(1,1)}^+ + \mathcal{S}_{(1,1)}^- , \\
\overline{\mathcal{V}}_{(2)} &= \mathcal{V}_{(1,1)} , \\
\overline{\mathcal{S}}_{(1)} &= \mathcal{S}_{(0,1)}^+ + \mathcal{S}_{(0,1)}^- + 2 \cdot \mathcal{V}_{(0,1)} + 2 \cdot \mathcal{T}_{(0,1)} + \mathcal{H}_{(0,1)} , \\
\overline{\mathcal{V}}_{(1)} &= \mathcal{V}_{(0,1)} + 2 \cdot \mathcal{H}_{(0,1)} .
\end{aligned}
\tag{A.4}$$

\mathcal{N}	spin	mass	content
(2, 2)	2		$\mathcal{G}_{(2,2)} = 1 \cdot [2] + 4 \cdot [\frac{3}{2}]^+ + 4 \cdot [\frac{3}{2}]^- + 16 \cdot [1]$ $+ 20 \cdot [\frac{1}{2}]^+ + 20 \cdot [\frac{1}{2}]^- + 25 \cdot [0]$ $+ 5 \cdot [t_2]^+ + 5 \cdot [t_2]^-$
(0, 2)	2		$\mathcal{G}_{(0,2)} = 1 \cdot [2] + 4 \cdot [\frac{3}{2}]^- + 5 \cdot [t_2]^-$
(0, 2)	3/2		$\mathcal{S}_{(0,2)} = 1 \cdot [\frac{3}{2}]^+ + 4 \cdot [1] + 5 \cdot [\frac{1}{2}]^-$
(0, 2)	1		$\mathcal{T}_{(0,2)} = 1 \cdot [t_2]^+ + 4 \cdot [\frac{1}{2}]^+ + 5 \cdot [0]$
(1, 1)	2		$\mathcal{G}_{(1,1)} = 1 \cdot [2] + 2 \cdot [\frac{3}{2}]^+ + 2 \cdot [\frac{3}{2}]^- + 4 \cdot [1]$ $+ 2 \cdot [\frac{1}{2}]^+ + 2 \cdot [\frac{1}{2}]^- + 1 \cdot [0]$ $+ 1 \cdot [t_2]^+ + 1 \cdot [t_2]^-$
(1, 1)	3/2		$\mathcal{S}_{(1,1)}^\pm = 1 \cdot [\frac{3}{2}]^\pm + 2 \cdot [1] + 4 \cdot [\frac{1}{2}]^\pm + 1 \cdot [\frac{1}{2}]^\mp$ $+ 2 \cdot [0] + 2 \cdot [t_2]^\pm$
(1, 1)	1		$\mathcal{V}_{(1,1)} = 1 \cdot [1] + 2 \cdot [\frac{1}{2}]^+ + 2 \cdot [\frac{1}{2}]^- + 4 \cdot [0]$
2	3/2	short	$\overline{\mathcal{S}}_{(2)} = 1 \cdot \overline{[\frac{3}{2}]} + 2 \cdot \overline{[1]} + 4 \cdot \overline{[\frac{1}{2}]} + 2 \cdot \overline{[0]} + 2 \cdot \overline{[t_2]}$
2	1	short	$\overline{\mathcal{V}}_{(2)} = 1 \cdot \overline{[1]} + 2 \cdot \overline{[\frac{1}{2}]} + 3 \cdot \overline{[0]}$
(0, 1)	2		$\mathcal{G}_{(0,1)} = 1 \cdot [2] + 2 \cdot [\frac{3}{2}]^- + 1 \cdot [t_2]^-$
(0, 1)	3/2		$\mathcal{S}_{(0,1)}^+ = 1 \cdot [\frac{3}{2}]^+ + 2 \cdot [1] + 1 \cdot [\frac{1}{2}]^-$
(0, 1)	3/2		$\mathcal{S}_{(0,1)}^- = 1 \cdot [\frac{3}{2}]^- + 2 \cdot [t_2]^-$
(0, 1)	1		$\mathcal{V}_{(0,1)} = 1 \cdot [1] + 2 \cdot [\frac{1}{2}]^-$
(0, 1)	0		$\mathcal{H}_{(0,1)} = 1 \cdot [\frac{1}{2}]^+ + 2 \cdot [0]$
(0, 1)	1		$\mathcal{T}_{(0,1)} = 1 \cdot [t_2]^+ + 2 \cdot [\frac{1}{2}]^+ + 1 \cdot [0]$
1	3/2	long	$\overline{\mathcal{S}}_{(1)} = 1 \cdot \overline{[\frac{3}{2}]} + 2 \cdot \overline{[1]} + 4 \cdot \overline{[\frac{1}{2}]} + 2 \cdot \overline{[0]} + 2 \cdot \overline{[t_2]}$
1	1	long	$\overline{\mathcal{V}}_{(1)} = 1 \cdot \overline{[1]} + 2 \cdot \overline{[\frac{1}{2}]} + 3 \cdot \overline{[0]}$

Table 8: Supergravity multiplets in $D = 6$. The first column shows the amount of supersymmetry, the second column indicates the maximal spin of the multiplet, the third column specifies whether the fields are massless (no indication) or massive (long or short).

A.3 Supergravity in $D = 4$

In this section we collect some information about the multiplet structure in four dimensions. The on-shell degrees of freedom of the various fields are summarized as follows

name	symbol	on-shell d.o.f.	(A.5)
massless spin 2	$[2]$	2_b	
massless spin 3/2	$[\frac{3}{2}]$	2_f	
massless spin 1	$[1]$	2_b	
massless spin 1/2	$[\frac{1}{2}]$	2_f	
massless spin 0	$[0]$	1_b	
massive spin 3/2	$\overline{[\frac{3}{2}]}$	4_f	
massive spin 1	$\overline{[1]}$	3_b	
massive spin 1/2	$\overline{[\frac{1}{2}]}$	2_f	
massive spin 0	$\overline{[0]}$	1_b	

In table 9 on pages 32 and 33 the massless and massive multiplets in four dimensions are summarized. This data has been taken from [40] and has been included here for completeness.

\mathcal{N}	spin	mass	content
8	2		$\mathcal{G}_{(8)} = 1 \cdot [2] + 8 \cdot [\frac{3}{2}] + 28 \cdot [1] + 56 \cdot [\frac{1}{2}] + 70 \cdot [0]$
6	2		$\mathcal{G}_{(6)} = 1 \cdot [2] + 6 \cdot [\frac{3}{2}] + 16 \cdot [1] + 26 \cdot [\frac{1}{2}] + 30 \cdot [0]$
6	3/2		$\mathcal{S}_{(6)} = 1 \cdot [\frac{3}{2}] + 6 \cdot [1] + 15 \cdot [\frac{1}{2}] + 20 \cdot [0]$
6	3/2	$\frac{1}{2}$ BPS	$\overline{\mathcal{S}}_{(6)} = 2 \cdot \overline{[\frac{3}{2}]} + 12 \cdot \overline{[1]} + 28 \cdot \overline{[\frac{1}{2}]} + 28 \cdot \overline{[0]}$
5	2		$\mathcal{G}_{(5)} = 1 \cdot [2] + 5 \cdot [\frac{3}{2}] + 10 \cdot [1] + 11 \cdot [\frac{1}{2}] + 10 \cdot [0]$
5	3/2		$\mathcal{S}_{(5)} = 1 \cdot [\frac{3}{2}] + 6 \cdot [1] + 15 \cdot [\frac{1}{2}] + 20 \cdot [0]$
5	3/2	$\frac{2}{5}$ BPS	$\overline{\mathcal{S}}_{(5)} = 2 \cdot \overline{[\frac{3}{2}]} + 12 \cdot \overline{[1]} + 28 \cdot \overline{[\frac{1}{2}]} + 28 \cdot \overline{[0]}$
4	2		$\mathcal{G}_{(4)} = 1 \cdot [2] + 4 \cdot [\frac{3}{2}] + 6 \cdot [1] + 4 \cdot [\frac{1}{2}] + 2 \cdot [0]$
4	3/2		$\mathcal{S}_{(4)} = 1 \cdot [\frac{3}{2}] + 4 \cdot [1] + 7 \cdot [\frac{1}{2}] + 8 \cdot [0]$
4	3/2	$\frac{1}{4}$ BPS	$\overline{\mathcal{S}}_{(4)}^{1/4} = 2 \cdot \overline{[\frac{3}{2}]} + 12 \cdot \overline{[1]} + 28 \cdot \overline{[\frac{1}{2}]} + 28 \cdot \overline{[0]}$
4	3/2	$\frac{1}{2}$ BPS	$\overline{\mathcal{S}}_{(4)}^{1/2} = 2 \cdot \overline{[\frac{3}{2}]} + 8 \cdot \overline{[1]} + 12 \cdot \overline{[\frac{1}{2}]} + 8 \cdot \overline{[0]}$
4	1		$\mathcal{V}_{(4)} = 1 \cdot [1] + 4 \cdot [\frac{1}{2}] + 6 \cdot [0]$
4	1	$\frac{1}{2}$ BPS	$\overline{\mathcal{V}}_{(4)} = 2 \cdot \overline{[1]} + 8 \cdot \overline{[\frac{1}{2}]} + 10 \cdot \overline{[0]}$
3	2		$\mathcal{G}_{(3)} = 1 \cdot [2] + 3 \cdot [\frac{3}{2}] + 3 \cdot [1] + 1 \cdot [\frac{1}{2}]$
3	3/2		$\mathcal{S}_{(3)} = 1 \cdot [\frac{3}{2}] + 3 \cdot [1] + 3 \cdot [\frac{1}{2}] + 2 \cdot [0]$
3	3/2	long	$\overline{\mathcal{S}}_{(3)}^l = 1 \cdot \overline{[\frac{3}{2}]} + 6 \cdot \overline{[1]} + 14 \cdot \overline{[\frac{1}{2}]} + 14 \cdot \overline{[0]}$
3	3/2	$\frac{1}{3}$ BPS	$\overline{\mathcal{S}}_{(3)}^{1/3} = 2 \cdot \overline{[\frac{3}{2}]} + 8 \cdot \overline{[1]} + 12 \cdot \overline{[\frac{1}{2}]} + 8 \cdot \overline{[0]}$
3	1		$\mathcal{V}_{(3)} = 1 \cdot [1] + 4 \cdot [\frac{1}{2}] + 6 \cdot [0]$
3	1	$\frac{1}{3}$ BPS	$\overline{\mathcal{V}}_{(3)} = 2 \cdot \overline{[1]} + 8 \cdot \overline{[\frac{1}{2}]} + 10 \cdot \overline{[0]}$
2	2		$\mathcal{G}_{(2)} = 1 \cdot [2] + 2 \cdot [\frac{3}{2}] + 1 \cdot [1]$
2	3/2		$\mathcal{S}_{(2)} = 1 \cdot [\frac{3}{2}] + 2 \cdot [1] + 1 \cdot [\frac{1}{2}]$
2	3/2	long	$\overline{\mathcal{S}}_{(2)}^l = 1 \cdot \overline{[\frac{3}{2}]} + 4 \cdot \overline{[1]} + 6 \cdot \overline{[\frac{1}{2}]} + 4 \cdot \overline{[0]}$
2	3/2	$\frac{1}{2}$ BPS	$\overline{\mathcal{S}}_{(2)}^{1/2} = 2 \cdot \overline{[\frac{3}{2}]} + 4 \cdot \overline{[1]} + 2 \cdot \overline{[\frac{1}{2}]}$

2	1		$\mathcal{V}_{(2)} = 1 \cdot [1] + 2 \cdot [\frac{1}{2}] + 2 \cdot [0]$
2	1	long	$\overline{\mathcal{V}}_{(2)}^l = 1 \cdot \overline{[1]} + 4 \cdot \overline{[\frac{1}{2}]} + 5 \cdot \overline{[0]}$
2	1	$\frac{1}{2}$ BPS	$\overline{\mathcal{V}}_{(2)}^{1/2} = 2 \cdot \overline{[1]} + 4 \cdot \overline{[\frac{1}{2}]} + 2 \cdot \overline{[0]}$
2	1/2		$\mathcal{H}_{(2)} = 2 \cdot [\frac{1}{2}] + 4 \cdot [0]$
2	1/2	$\frac{1}{2}$ BPS	$\overline{\mathcal{H}}_{(2)} = 2 \cdot \overline{[\frac{1}{2}]} + 4 \cdot \overline{[0]}$
1	2		$\mathcal{G}_{(1)} = 1 \cdot [2] + 1 \cdot [\frac{3}{2}]$
1	3/2		$\mathcal{S}_{(1)} = 1 \cdot [\frac{3}{2}] + 1 \cdot [1]$
1	3/2	long	$\overline{\mathcal{S}}_{(1)}^l = 1 \cdot \overline{[\frac{3}{2}]} + 2 \cdot \overline{[1]} + 1 \cdot \overline{[\frac{1}{2}]}$
1	1		$\mathcal{V}_{(1)} = 1 \cdot [1] + 1 \cdot [\frac{1}{2}]$
1	1	long	$\overline{\mathcal{V}}_{(1)}^l = 1 \cdot \overline{[1]} + 2 \cdot \overline{[\frac{1}{2}]} + 1 \cdot \overline{[0]}$
1	1/2		$\mathcal{C}_{(1)} = 1 \cdot [\frac{1}{2}] + 2 \cdot [0]$
1	1/2	long	$\overline{\mathcal{C}}_{(1)} = 1 \cdot \overline{[\frac{1}{2}]} + 2 \cdot \overline{[0]}$

Table 9: Supergravity multiplets in $D = 4$. The first column shows the amount of supersymmetry, the second column indicates the maximal spin of the multiplet, the third column specifies whether the fields are massless (no indication) or massive (long or shortened). For more details see [40].

References

- [1] K. S. Narain, M. H. Sarmadi, and C. Vafa, “Asymmetric Orbifolds,” *Nucl. Phys.* **B288** (1987) 551.
- [2] I. Antoniadis, C. Bachas, C. Kounnas, and P. Windey, “Supersymmetry Among Free Fermions and Superstrings,” *Phys. Lett.* **B171** (1986) 51–56.
- [3] I. Antoniadis, C. P. Bachas, and C. Kounnas, “Four-Dimensional Superstrings,” *Nucl. Phys.* **B289** (1987) 87.
- [4] S. Ferrara and C. Kounnas, “Extended Supersymmetry in Four-dimensional Type II Strings,” *Nucl. Phys.* **B328** (1989) 406–438.
- [5] I. Brunner, A. Rajaraman, and M. Rozali, “D-branes on asymmetric orbifolds,” *Nucl. Phys.* **B558** (1999) 205–215, [hep-th/9905024](#).
- [6] M. Gutperle, “NonBPS D-branes and enhanced symmetry in an asymmetric orbifold,” *JHEP* **08** (2000) 036, [hep-th/0007126](#).
- [7] M. Bianchi, “Bound-states of D-branes in L-R asymmetric superstring vacua,” *Nucl. Phys.* **B805** (2008) 168–181, [0805.3276](#).
- [8] M. Bianchi, “From Twists and Shifts to L-R asymmetric D-branes,” *Fortsch. Phys.* **57** (2009) 356–366, [0902.0650](#).
- [9] A. Dabholkar and C. Hull, “Duality twists, orbifolds, and fluxes,” *JHEP* **09** (2003) 054, [hep-th/0210209](#).
- [10] S. Hellerman, J. McGreevy, and B. Williams, “Geometric constructions of nongeometric string theories,” *JHEP* **01** (2004) 024, [hep-th/0208174](#).
- [11] A. Flournoy, B. Wecht, and B. Williams, “Constructing nongeometric vacua in string theory,” *Nucl. Phys.* **B706** (2005) 127–149, [hep-th/0404217](#).
- [12] A. Flournoy and B. Williams, “Nongeometry, duality twists, and the worldsheet,” *JHEP* **01** (2006) 166, [hep-th/0511126](#).
- [13] S. Hellerman and J. Walcher, “Worldsheet CFTs for Flat Monodrofolds,” [hep-th/0604191](#).
- [14] C. Condeescu, I. Florakis, and D. Lüster, “Asymmetric Orbifolds, Non-Geometric Fluxes and Non-Commutativity in Closed String Theory,” *JHEP* **04** (2012) 121, [1202.6366](#).

- [15] C. Condeescu, I. Florakis, C. Kounnas, and D. Lüst, “Gauged supergravities and non-geometric Q/R-fluxes from asymmetric orbifold CFT’s,” *JHEP* **10** (2013) 057, 1307.0999.
- [16] R. Blumenhagen, M. Fuchs, and E. Plauschinn, “Partial SUSY Breaking for Asymmetric Gepner Models and Non-geometric Flux Vacua,” 1608.00595.
- [17] G. Aldazabal, D. Marques, and C. Nunez, “Double Field Theory: A Pedagogical Review,” *Class. Quant. Grav.* **30** (2013) 163001, 1305.1907.
- [18] D. S. Berman and D. C. Thompson, “Duality Symmetric String and M-Theory,” *Phys. Rept.* **566** (2014) 1–60, 1306.2643.
- [19] O. Hohm, D. Lüst, and B. Zwiebach, “The Spacetime of Double Field Theory: Review, Remarks, and Outlook,” *Fortsch. Phys.* **61** (2013) 926–966, 1309.2977.
- [20] D. Gepner, “Space-Time Supersymmetry in Compactified String Theory and Superconformal Models,” *Nucl. Phys.* **B296** (1988) 757.
- [21] D. Gepner, “Exactly Solvable String Compactifications on Manifolds of $SU(N)$ Holonomy,” *Phys. Lett.* **B199** (1987) 380–388.
- [22] J. Fuchs, A. Klemm, C. Scheich, and M. G. Schmidt, “Gepner Models With Arbitrary Affine Invariants and the Associated Calabi-yau Spaces,” *Phys. Lett.* **B232** (1989) 317–322.
- [23] J. Fuchs, A. Klemm, C. Scheich, and M. G. Schmidt, “Spectra and Symmetries of Gepner Models Compared to Calabi-yau Compactifications,” *Annals Phys.* **204** (1990) 1–51.
- [24] A. N. Schellekens and S. Yankielowicz, “Extended Chiral Algebras and Modular Invariant Partition Functions,” *Nucl. Phys.* **B327** (1989) 673–703.
- [25] A. N. Schellekens and S. Yankielowicz, “Modular Invariants From Simple Currents: An Explicit Proof,” *Phys. Lett.* **B227** (1989) 387–391.
- [26] A. N. Schellekens and S. Yankielowicz, “New Modular Invariants for $N = 2$ Tensor Products and Four-dimensional Strings,” *Nucl. Phys.* **B330** (1990) 103–123.
- [27] B. Gato-Rivera and A. N. Schellekens, “Asymmetric Gepner Models: Revisited,” *Nucl. Phys.* **B841** (2010) 100–129, 1003.6075.
- [28] B. Gato-Rivera and A. N. Schellekens, “Asymmetric Gepner Models II. Heterotic Weight Lifting,” *Nucl. Phys.* **B846** (2011) 429–468, 1009.1320.

- [29] D. Israel and V. Thiéry, “Asymmetric Gepner models in type II,” *JHEP* **02** (2014) 011, 1310.4116.
- [30] D. Israel, “Nongeometric Calabi-Yau compactifications and fractional mirror symmetry,” *Phys. Rev.* **D91** (2015) 066005, 1503.01552. [Erratum: *Phys. Rev.* **D91**, no.12, 129902 (2015)].
- [31] R. Blumenhagen and A. Wisskirchen, “Exactly solvable (0,2) supersymmetric string vacua with GUT gauge groups,” *Nucl. Phys.* **B454** (1995) 561–586, hep-th/9506104.
- [32] R. Blumenhagen, R. Schimmrigk, and A. Wisskirchen, “The (0,2) exactly solvable structure of chiral rings, Landau-Ginzburg theories, and Calabi-Yau manifolds,” *Nucl. Phys.* **B461** (1996) 460–492, hep-th/9510055.
- [33] R. Blumenhagen, R. Schimmrigk, and A. Wisskirchen, “(0,2) mirror symmetry,” *Nucl. Phys.* **B486** (1997) 598–628, hep-th/9609167.
- [34] L. J. Dixon, V. Kaplunovsky, and C. Vafa, “On Four-Dimensional Gauge Theories from Type II Superstrings,” *Nucl. Phys.* **B294** (1987) 43–82.
- [35] R. Blumh, L. Dolan, and P. Goddard, “Unitarity and Modular Invariance as Constraints on Four-dimensional Superstrings,” *Nucl. Phys.* **B309** (1988) 330–360.
- [36] P. Anastasopoulos, M. Bianchi, J. F. Morales, and G. Pradisi, “(Unoriented) T-folds with few T’s,” *JHEP* **06** (2009) 032, 0901.0113.
- [37] S. Deser and B. Zumino, “Broken Supersymmetry and Supergravity,” *Phys. Rev. Lett.* **38** (1977) 1433–1436.
- [38] E. Cremmer, B. Julia, J. Scherk, P. van Nieuwenhuizen, S. Ferrara, and L. Girardello, “Super-higgs effect in supergravity with general scalar interactions,” *Phys. Lett.* **B79** (1978) 231–234.
- [39] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello, and P. van Nieuwenhuizen, “Spontaneous Symmetry Breaking and Higgs Effect in Supergravity Without Cosmological Constant,” *Nucl. Phys.* **B147** (1979) 105.
- [40] L. Andrianopoli, R. D’Auria, S. Ferrara, and M. A. Lledo, “Super Higgs effect in extended supergravity,” *Nucl. Phys.* **B640** (2002) 46–62, hep-th/0202116.
- [41] A. Dabholkar and J. A. Harvey, “String islands,” *JHEP* **02** (1999) 006, hep-th/9809122.

- [42] <http://wwwth.mpp.mpg.de/members/blumenha/Examples.zip>.
- [43] R. Blumenhagen and E. Plauschinn, “Introduction to conformal field theory,” *Lect. Notes Phys.* **779** (2009) 1–256.
- [44] C. Angelantonj and A. Sagnotti, “Open strings,” *Phys. Rept.* **371** (2002) 1–150, [hep-th/0204089](#). [Erratum: *Phys. Rept.* 376, no. 6, 407 (2003)].
- [45] R. D’Auria, S. Ferrara, and C. Kounnas, “ $N = (4, 2)$ chiral supergravity in six-dimensions and solvable Lie algebras,” *Phys. Lett.* **B420** (1998) 289–299, [hep-th/9711048](#).
- [46] J. A. Strathdee, “Extended Poincare Supersymmetry,” *Int. J. Mod. Phys.* **A2** (1987) 273.